Contributed Session 2: ODEs and Applications

Nonlinear boundary value problems for second order differential equations describing concave equilibrium capillary surfaces

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In this paper the following boundary value problems are considered:

$$\begin{cases} z_e'' = \dfrac{\rho \cdot g \cdot z_e - p_e}{\gamma} \left[1 + (z_e')^2\right]^{\frac{3}{2}} - \dfrac{1}{r} \left[1 + (z_e')^2\right] z_e' \\ z_e'(r_e) = -\tan\left(\frac{\pi}{2} - \alpha_g\right) \\ z_e'(R_{ge}) = -\tan \alpha_c \\ z_e(R_{ge}) = 0, \;\; z_e(r) \text{ strictly decreasing on } [r_e, R_{ge}] \end{cases}$$

$$\begin{cases} z_i'' = \dfrac{\rho \cdot g \cdot z_i - p_i}{\gamma} \left[1 + (z_i')^2\right]^{\frac{3}{2}} - \dfrac{1}{r} \left[1 + (z_i')^2\right] z_i' \\ z_i'(r_i) = \tan\left(\frac{\pi}{2} - \alpha_g\right) \\ z_i'(R_{gi}) = \tan \alpha_c \\ z_i(R_{gi}) = 0, \;\; z_i(r) \text{ strictly increasing on } [R_{gi}, r_i] \end{cases}$$

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Dynamically consistent nonstandard finite difference schemes for continuous dynamical systems

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Dynamic consistency of a numerical scheme for a given continuous dynamical system generally means that the numerical solutions have the same dynamic behavior as the solutions of the continuous system. We give precise definition of this concept in the following way. Let a dynamical system be given by a system of equations of the form

$$\frac{dy}{dt} = f(y), \;\; y(0) = x, \tag{1}$$

where $f \in C^1(D, D)$, $D \subset \mathbb{R}^d$, and the solution $y(x,t)$ is assumed to exists for all $t \in [0, +\infty)$ and $x \in D$. The system (1) is approximated on a time-grid with step-size h by a difference equation of the form

$$y_{k+1} = F(h, y_k), \;\; y_0 = x, \tag{2}$$

where $y_k \approx y(x, t_k)$. Denote by $S(t)$ the evolution operator associated with (1), that is, $S(t)x = y(x,t)$, $t > 0$, $x \in D$, and let $\mathcal{F}(h) : D \to D$ be the mapping $\mathcal{F}(h)(y) = F(h, y)$. Then the scheme (2) is dynamically consistent with (1) if $\mathcal{F}(h)$ and $S(t)$ are topologically conjugate for every $t > 0$ and $h > 0$, that is, there exists a homeomorphism $\phi : D \to D$ such that $\mathcal{F}(h) \circ \phi = \phi \circ S(t)$.

Numerical schemes for particular systems in one or more dimensions constructed via the nonstandard finite method will be discussed.

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On the constants of the motion of the generalized Ermakove systems

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Physical problems such as the Kepler and MICZ problems are related systems in that they both possess a common subalgebra $sl(2,R) \oplus so(3)$. In purview of this, we consider the system of the most general system

$$\ddot{\mathbf{r}} = \frac{1}{r^3} \left\{ C_1(L)\,\hat{\mathbf{r}} + C_2(L)\,\hat{\mathbf{w}} + C_3(L)\,\hat{\mathbf{L}} \right\}$$

invariant under $sl(2,R) \oplus so(3)$ and remark on the influence of each of its symmetry generators on its constants of the motion.

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Numerical solution of linear differential equations with fractional order

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Definition of fractional derivative and fractional integral are given. Results on the qualitative behavior of solution of linear differential equations of fractional order are cited. The collocation method with orthogonal polynomials is applied. The resulting numerical scheme is discussed and numerical examples are given to illustrate our results.

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Symmetry Breaking due to Coupling in systems with Symmetry

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In this work we consider low-dimensional ODEs with symmetry coupled with other degrees of freedom. Due to the coupling the original symmetry is broken. Nevertheless, a strong signature of the original symmetry is still present in coupled systems for small to intermediate strength of coupling. In particular, preferred directions and local basins of attraction appear in coupled systems.

We utilize the stochastic mode-reduction strategy to explain this phenomena. In particular, the preferred directions are related to the eigendirections of the resulting damping and diffusion matrices. The eigendirections do not depend on the coupling strength and predict the appearance of local basins of attraction for a wide range of parameters.

Several low-dimensional examples with different symmetry properties will be utilized to illustrate the phenomena.

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Numerical Methods for Optimization-Constrained Differential Equations with Discontinuities

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Dynamic optimization problems arise when coupling a global optimization problem with constraints and ordinary differential equations.

We present a numerical approach for the resolution of systems of optimization-constrained differential equations arising in atmospheric chemistry, when modeling the dynamics of organic aerosol particles.

We consider a global optimization problem with inequality constraints and we couple differential equations with its first order optimality conditions. A system of differential-algebraic equations of index one under constraints is obtained. The numerical resolution of based on a second order implicit time discretization and a primal-dual interior-point method coupled to a Newton iteration.

The activation/deactivation of the inequality constraints induce discontinuities in the time evolution of the variables and bifurcations between local and global minima. We propose event location techniques based on extrapolation methods to accurately track the discontinuity points. Due to the coupling with the optimization problem, the switching functions describing the discontinuities are not known explicitly.

Numerical results are presented for the computation of the evolution and phase transitions in organic atmospheric aerosol particles. We finally show the extension to systems of differential equations constrained by sequences of optimization problems for the modeling of populations of particles.

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Nilpotent Period-Doubling Bifurcations for Three-Cell Homogeneous Coupled Maps

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Network architecture can lead to robust synchrony in coupled maps and to codimension one bifurcations from synchronous fixed-points at which the associated Jacobian is nilpotent.

I will discuss the codimension one synchrony-breaking nilpotent period-doubling bifurcations for three-cell coupled maps. Interesting phenomena occur for all these coupled maps — a branch of 2-period points with amplitude growing as $|\lambda|^{\frac{1}{6}}$ for coupled networks with feed-forward feature, as well as *multiple* (*two*) branches of 2-period points with amplitude growing as $|\lambda|^{\frac{1}{2}}$ for coupled networks without feed-forward feature.

I will also discuss some results related to patterns of synchrony translate from coupled maps

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Spiral Classical Motion in Angular Potentials

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A smooth real-valued function V on $\mathbb{R}^n \setminus \{0\}$, $n \geq 2$, is called angular potential or homogeneous potential of degree zero if it is independent of $|\mathbf{r}|$. Let $\mathbf{Cr} = \{\omega \in S^{n-1} : \nabla V(\omega) = 0\}$. A standard result for the motion of a classical particle in two dimensional angular potentials states that if $\mathbf{r}(\cdot) : (0, \infty) \to \mathbb{R}^2 \setminus \{0\}$ is a solution to Newton's equation $\ddot{\mathbf{r}}(t) = -\nabla V(\mathbf{r})$ and $\omega(t) = \mathbf{r}(t)/|\mathbf{r}(t)|$, then $\lim_{t\to\infty} \omega(t)$ exists and belongs to $\mathbf{Cr}$. This fact was established by Herbst and he conjectured a similar result for general angular potentials in dimension $n \geq 3$. In this talk we present an example of an angular potential in three dimensions, such that the related Newton's equation has a solution for which $\omega(t)$ spirals towards the equator of S^2, and thus $\lim_{t\to\infty} \omega(t)$ does not exist.

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Constrained Maximum Likelihood Estimation of Stochastic Volatility Models

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In mathematical option pricing, prices and volatility of stocks are often first modelized by a pair of coupled stochastic differential equations, such as the well known Heston model.

We develop and implement an efficient algorithm to estimate the 5 parameters of Heston models from arbitrary given series of joint observations for stock prices and volatility. First we approximate the log-likelihood through Euler discretization, with a precise control of the accuracy in approximation.

Contrary to several other published methods, we consider that the time interval T separating two observations is unknown and must also be estimated from the data, so that we actually estimate 6 parameters with a clear gain in accuracy of the fit.

We implement an original type of constrained maximization of the approximate log-likelihood, making sure that the non linear constraints guaranteeing a good accuracy of the log-likelihood approximation are satisfied.

We then perform fast Monte Carlo simulations of the fitted Heston model, to compute approximate variances of the 6 errors of estimation. We study the asymptotic behavior of estimation errors . We apply this method to S&P 500 index prices with volatility data extracted from the Chicago Board of Exchange (CBOE) volatility index (VIX).

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A Nonlinear Delay-Differential Equation with Harmonic Excitation

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Based on substantial theoretical and experimental evidence it is now widely-accepted that the so-called regenerative effect plays a prominent role in machine tool dynamics. Because of some disturbances the tool starts a damped oscillation relative to the workpiece, making its surface uneven. The cutting force is a function of the chip geometry and in turn the chip geometry is determined by the tool position and the surface of the workpiece. However, the surface of the workpiece is produced by the same tool on the rotating workpiece, hence the cutting force also depends on the tool position one revolution ago. This dependence on the tool position in the past leads to a mathematical description of the model as a delay-differential equation. The machining tool can be subject to different kinds of excitations. The forcing may have external sources (such as rotating imbalance or misalignment of the workpiece) or it can arise from the cutting process itself (e.g. chip formation). Here we investigate the classical 1-DOF tool vibration model with cutting force nonlinearity and periodic forcing.

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Center Manifold Analysis of the Delayed Lienard Equation

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In this paper, we analyze the delayed Lienard equation

$$\ddot{x}(t) + f(x(t))\dot{x} + g(x(t-\tau)) = 0, \qquad (1)$$

where $f, g \in C^4, f(0) > 0, g(0) = 0, g'(0) = 1$, $\tau > 0$ is a finite time-delay. The existence of Hopf bifurcation is shown using center manifold reduction and the Poincare-Lyapunov constant that determines the criticality of the bifurcation is calculated. Earlier studies neglected terms due to the curvature of the manifold. Several examples are given to compare our theoretical analysis and numerical results using DDE-Biftool and a novel least-squares numerical technique for the calculation of the Poincare-Lyapunov constant.

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Bio-Inspired "Dynamically Adjusted Traffic Rate Alterations v2" (DATRAv2)

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“No Man is an Island” [John Donne], all types of networks have a fundamental role in humanity, and their failure often causes severe adverse consequences. We address the critical issue of network failures, through our model, DATRAv2 with the aim of understanding the cause and preventing the occurrence of such failures. Our simulations are applied to graphs with built–in attributes (buffer capacity, service power etc.), which stay constant during each simulation. We implement simulations, which are portable to any type of topology and amount of initial traffic, aiming to reach stationary flow at each network node as an optimum point, when it is not perturbed by external factors. DATRAv2 achieves stationary flow within each node significantly more quickly than our first version of DATRA. Our model is investigated through a series of simulations on multiple nodes forming diverse graphs. The simulations are based on our algorithm, Networking–Runge–Kutta (NRK), which initially calculates the number of coupled ODEs that describe the state of the studied network node. Thereafter, it utilizes the well known Runge-Kutta algorithm, which is adjusted, in order to integrate the principle of a graph by including an updated transition matrix at each time step of the numerical analysis.

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Fixed Point Theorems and Fundamental Theorem of Calculus

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Some new Fixed Point Results will be discussed. Also, the Fundamental Theorem of Calculus (FTC) will be discussed. The (FTC) involving the Riemann Integral is one of the most pervasive and basic results in Real Analysis that almost every student of Mathematics has been exposed to and has extensively used in great variety of areas of Mathematics. FTC appears in every book on Calculus and Real Analysis, however, with an ever-present gap. It seems that nowhere in the literature one can find a nontrivial and easily accessible to the reader an "if and only if" Theorem of FTC involving the Riemann Integral. The following is intended to eliminate this centuries old ubiquitous gap.

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Group Classification of a generalized Lane-Emden-type equation

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We perform the Lie group classification of a generalized Lane-Emden-type equation, which arises in several applications. These applications include the theory of stellar structure, the thermal behaviour of a spherical cloud of gas, isothermal gas spheres and theory of thermionic currents. It is found that the equivalence algebra is three-dimensional and in a special case four-dimensional. Various cases for which the principal Lie algebra extends are obtained.

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The algebraic structure of Abelian integrals for a kind of purterbated quartic Hamoltonian systems

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In this paper, we obtain the algebraic expression of Abelian ingetrals

$$I(h) = \oint_{\Gamma_h} f(x,y)dx - g(x,y)dy$$

for a kind of purterbated qrartic Hamiltonian systems with Hamilton function

$$H(x,y) = x^2 + y^2 + ax^4 + bx^2y^2 + cy^4,$$

where Γ_h is the oval defined by

$$H(x,y) = h, h \in (0, h_1)$$

.By this result, we get the upper bound of number of zereo for some special systems, which is important in the studies of weaked Hilbert's 16th problem.

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QUALITATIVE EQUIVALENCE BEHAVIOR OF FIRST ORDER AUTONOMOUS TRIGONOMETRIC ODEs

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This paper presents results on the qualitative equivalence behavior of first order autonomous ordinary differential equations of the form $x^{/} = f(x)$ where f(x) is a trigonometric function. Since f(x) is necessarily an infinite series, it is approximated as a polynomial function by considering the first few terms of its power series expansion. This allows for the calculation of the number of qualitative classes of the set $\{x^{/} = f(x)\}$ using classical method based on the nature of the phase portraits of $x^{/} = f(x)$. The paper essentially focusses on the trigonometric functions sin ax and cos ax, where a is a constant taken, without any loss of generality, to be one. The qualitative behavior of $x^{/} = f(x)$ when f(x) is trigonometric is then compared to that when f(x) is hyperbolic.The qualitative properties relevant to this work are existence and uniqueness of solutions.

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Fučik type spectra for essentially nonlinear equations

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Fučik spectra appear when considering equations of the type $x'' = -\lambda x^+ + \mu x^-$ together with some (mostly two-point) boundary conditions, $x^\pm = \max\{\pm x, 0\}$. We study the problem $x'' = -\lambda f(x^+) + \mu g(x^-)$, $x(a) = 0$, $x(b) = 0$, where f and g are nonlinear positive valued functions, $f(0) = g(0) = 0$. To avoid continuous spectra (filling all the plane of parameters (λ, μ)), we impose the normalization condition. Descriptions of spectra are given and peculiar features (which are not observable in a piece-wise linear case) are discussed.

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Multiple solutions of the second order nonlinear boundary value

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We consider nonlinear boundary value problems (BVP) of the type

$$(l_2x)(t) = f(t, x, x'),\ x(0) = 0,\ x(\pi) = 0,$$

where $(l_2x)(t)$ is the second order linear differential expression. We state results on the existence and multiplicity of solutions incuding the resonant case also. The method used is that of: F. Sadyrbaev, I. Yermachenko. Types of solutions and multiplicity results for second order nonlinear boundary value problems. *Discrete and Continuous Dynamical Systems-Supplement* **2007**, pp.1061 - 1069.

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Buckling of Graphene Sheets

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We investigate several buckling problems for elastica under non-local body forces. These problems model the interaction by van der Waals forces of graphene—a single-atom-thick sheet of carbon atoms—with a rigid substrate. We present a qualitative analysis of the buckling of graphene subject to edge loading. We also investigate how the spacing between the graphene and the substrate influences buckling. This work is motivated by recently developed techniques for isolating individual graphene layers and by the potential applications of graphene in nanoscale devices and in nanocomposites.

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