Contributed Session 9: PDEs and Applications

Patterns of two prey-one predator models with ratio-dependent predator influence

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In this talk, we discuss a 3-species model with two competing prey and one predator interaction with ratio-dependent functional response. First, the stability of all nonnegative equilibria is investigated for the system. Second, we study the non-constant positive steady -states of the systems. Furthermore, by considering the same model with a cross-diffusion, we discuss the effect of cross-diffusion to occurrence of patterns for the system.

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Estimation of effective system from discretely sampled data

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We discuss a procedure to identify a low-dimensional effective stochastic system from a large deterministic or stochastic system. These variables are selected apriori as a projection of the full dynamics or determined from physical considerations.

We use Maximum Likelihood techniques to estimate the effective model from the data corresponding to the selected variables. The likelihood function is approximated by an appropriate Gaussian distribution obtained from discretization.

It is observed that the stochastic mode-reduction strategy can be utilized to predict the functional form of the reduced model even in the regimes without scale-separation.

We present a variety of simple systems constructed by coupling Lorenz 63 model with basic stochastic processes to explain our method.

In our search for the optimal sampling step-size Δt we analyze the behavior of the estimates vis-a-vis the step-size and estimate order of convergence.

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Electrical Impedance Tomography Reconstructions in 3D via a Direct Method

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Electrical Impedance Tomography (EIT) is a relatively new method that creates an image of the interior of an unknown object by reconstructing the electrical conductivity distribution of that object from measurements of the voltage arising from an applied current on the boundary. The reconstruction of the conductivity distribution is mathematically challenging since it is an ill-posed inverse problem.

Most of the existing reconstruction methods are for the 2D geometry while fewer algorithms have been considered for three dimensional domains. Medical application in the 3D geometry include among others breast cancer detection and the location of centers of activity in the brain. In this talk I will introduce a direct method for three dimensional bounded domains based on the paper by A. Nachman 'Reconstruction from boundary measurements', Ann. of Math.,128, 1988 and propose an implementation for the reconstruction of spherically symmetric conductivities in a sphere. Computational results will be included.

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A System of Partial Differential Equations for multi-lane freeways

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Linear perturbation is used to develope a sytem of partial differential equations for densities of the lanes in freeway as a function of position and time. It is assumed that speed in each lane is a function of densities in all lanes and that the total number of vehicles is conserved.

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Unbounded Solutions of the Modified Korteweg-De Vries Equation

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Consider the initial value problem for the focusing mKdV equation $u_t + u^2 u_x + u_{xxx} = 0$ where the initial data u_0 is a smooth unbounded function having asymptotic expansion at infinity in decreasing powers of x. One may construct an asymptotic solution with polynomial coefficients if $\frac{d^n}{dx^n} u_0 = o(|x|^{\frac{1}{2}-n})$ for each $n \in \mathbb{N}$. We show that an asymptotic solution for the focusing mKdV differs from a genuine solution by a function in $C_t^\infty S_x(\mathbb{R} \times \mathbb{R})$ that solves a generalized version of the focusing mKdV equation. The latter equation is solved by discretization methods.

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Application of Minkowski space geometry in the study of some nonlinear equations

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The differential geometric approach to the study of integrability and solutions of some physically important nonlinear equations such as Nonlinear Schrodinger, modified Kortuweg-deVries, Belavin-Polyakov etc. are well known and is an active field of research. However for curve and surface solutions of the defocusing case of the equations, i.e. when the nonlinear term is negative, Minkowski space of three and four dimensions seems to be a more natural setting and we demonstrate this for a few standard equations. We show how the soluttion of the Manakov system, which is essentially a system of coupled Nonlinear Schrodinger equations has a nice curve representation in Minkowski 4-space.

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Chaotic scattering via the separatrix map in solitary wave interactions

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We present a new and complete analysis of the n-bounce resonance and chaotic scattering in solitary wave collisions. In these phenomena, the speed at which a wave exits a collision depends in a complicated fractal way on its input speed. We present a new asymptotic analysis of collective-coordinate ordinary differential equations (ODEs), reduced models that reproduce the dynamics of these systems. We reduce the ODEs to discrete-time iterated separatrix maps and obtain new quantitative results unraveling the fractal structure of the scattering behavior. These phenomena have been observed repeatedly in many solitary wave systems over 25 years.

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A technique for solving a class of hyperbolic equations with nonlocal conditions by Adomian method

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Abstract. In this paper, we show that with a few modifcations and techniques the Adomian Decomposition Method (ADM) for solving a class of hyperbolic equations with nonlocal conditions can be applied to obtain the exact solutions to this type of problems. The clue of this one consists in transforming the boundary value nonlocal problem into a classical problem whose solution is obtained.

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Error estimates on weak solutions to a non-equilibrium moving-boundary system modeling corrosion of materials

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Galerkin approximations to solutions of a two-phase one-dimensional non-equilibrium moving-boundary system describing the penetration of a sharp-reaction interface into a porous material are considered. We address the well-posedness of the model. Additionally, we prove a priori and a posteriori error estimates for both semi-discrete fields of active concentrations and position of the moving interface. The main feature of the problem is that the non-linear coupling of the PDE system is due to the non-linearity of surface and volume productions by reaction and of the moving interface itself.

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Stability of arbitrary-amplitude noncharacteristic boundary layers

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Using pointwise semigroup techniques of Zumbrun-Howard and Mascia-Zumbrun, we obtain sharp global pointwise Green function bounds for noncharacteristic boundary layers of arbitrary amplitude. These together with weighted energy estimates allow us to analyze linearized and nonlinearized stability of noncharacteristic boundary layers of one-dimensional hyperbolic-parabolic systems of conservation laws, showing that both are equivalent to a numerically checkable Evans function condition. [joint work with my advisor Kevin Zumbrun.] We also verify this spectral stability condition for isentropic gas dynamics case, yielding a complete classification of stability in the isentropic one-dimensional case. [joint work with N. Costanzino, J. Humpherys, and K. Zumbrun.]

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Dimensionality Reduction Using Markov Chain Modeling

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We present a method to reduce the dimensionality of a system by eliminating interactions with the nonessential variables and introducing a simple stochastic model for the right hand side (RHS) of the variables of interest. The goal of this approach is to reproduce the averaged behavior of these variables. The stochastic model is constructed by replacing an appropriate part of the right-hand side of the equations by a Markov chain. Therefore, the Markov chain represents interactions between the variables of interest and small scales. The properties of the Markov chain are estimated from a short simulation of the full model. In the full model, the time step in simulations is limited by the behavior of the small scale variables. The first advantage of this approach is that the dimensionality of the system can be reduced considerably. In addition, since the Markov chain can be constructed for an arbitrary time-step the effective equation can be integrated with a larger time step compared with the full model. We present several examples illustrating the approach and demonstrating that for a variety of deterministic and stochastic systems statistical properties of full models are in a good agreement with the statistical properties of simplified stochastic models.

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Conjugate points on the 3D volumorphism group

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I will discuss some results on the geometry of the group of volume-preserving diffeomorphisms of a compact manifold (the configuration space of an ideal fluid). Along a particular geodesic (a Lagrangian fluid flow), we can study the conjugate points, which give information about fluid stability and possibly singularities of a flow. For two-dimensional fluids, a result of Ebin-Misolek-Preston shows that the conjugate points are discrete along a geodesic, corresponding to the nicer geometric behavior of two-dimensional flows. In three-dimensions, they can be much more complicated. I will describe a recent result, using WKB analysis, that proves conjugate points along a three-dimensional flow generically occur in intervals.

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The immersed interface method for the interaction of a fluid with rigid solids

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A solid boundary immersed in a fluid can be formulated as a singular force in the Navier-Stokes equations. Two problems arise regarding this formulation. One is how to calculate the singular force. The other is how to treat the force singularity. The latter is solved in the immersed interface method with second-order accuracy and the sharp fluid-solid interface by incorporating singularity-induced flow jump conditions into discretization schemes. This talk focues on the former problem. In particular, I will present an explicit approach for computing the singular force to enforce prescribed motion of a rigid boundary in an incompressible viscous flow. This approach eliminates ad hoc penalty models for rigid boundaries and associated stiffness, which gives it numerical stability at relatively high Reynolds numbers. Simulations of circular Couette flow, flow past a cylinder, and flow around flappers will be shown to test the accuracy, stability, and efficiency of this approach.

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