

## Special Session 11: Hamiltonian systems and applications

Amadeu Delshams, Universitat Politècnica de Catalunya, Catalonia

Marian Gidea, Northeastern Illinois University, USA

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### Estimation of a hyperbolic invariant set in the three-body problem associated to Lyapunov orbits and resonance motions

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Marian Gidea and Francesco Topputo

We provide a method for explicitly estimating a hyperbolic invariant set in the restricted three-body problem based on local information only, using explicit formula, instead of using the global dynamics of the invariant manifolds associated to the Lyapunov orbits. This provides a simplified way to estimate this dynamically important region. We also discuss interesting resonance motions.

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### Numerical computation of KAM tori for equilibrium states in 1-D statistical mechanics models

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Rafael de la Llave

We will introduce methods for numerical computation of KAM quasi-periodic equilibrium solutions in statistical mechanics models. For our methods we use an extension of the Lagrangian proof of KAM for twist mappings. Then we will use these computations to estimate the breakup of quasi-periodic solutions.

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### Other topologies for topological conjugacy between diffeomorphisms of non-compact manifolds

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Let us consider the group of homeomorphisms (resp. diffeomorphisms)  $\text{Hom}(M)$  (resp.  $\text{Diff}(M)$ ), with  $M$  a non-compact finite dimensional manifold, and let us denote by  $\mathbb{G}$  one of the homeomorphisms or diffeomorphisms groups. We consider the conjugacy problem on  $\mathbb{G}$ , *i.e.* given  $f$  and  $g$  in  $\mathbb{G}$ , the existence problem of an homeomorphism  $h$  that solves  $f = h \circ g \circ h^{-1}$ ; from a new standpoint that relies essentially on building new topologies on  $\mathbb{G}$ , different from the standard compact-open topology or Whitney topology, with additional structures that permit to deal with the conjugacy equation via new methods and computations. More precisely, our strategy will consist of building semimetrics subgroups (*i.e.* in which the triangle inequality is not required) of  $\mathbb{G}$  that present some completeness properties by intersecting these subgroups with the set of  $\mathbb{G}$ -valued sequences which are bounded on every compact of  $M$ . On these subgroups, the inherited topology is finer than the compact-open topology and, of course, coarser than the Whitney topology. This framework provides, in some sense, an intermediate setting for dealing with the conjugacy problem on non-compact manifolds, as fully discussed and illustrated by Kotus, Krych and Nitecki (AMS, Vol. 37, No. 261, 1982) in their study of global structural stability of flows on open surfaces. Thanks to the construction developed here, along with its properties, we will expose how new fixed point theorem, for maps of  $\mathbb{G}$ , can be derived with as main application, the solving of the conjugacy problem without any kind of hyperbolicity assumption. In the framework used here, this last point will be treated via an iteration process which consists of solving a carefully chosen sequence of perturbed conjugacy equations along with control of the way of going to the limit. Motivated by physical considerations, we will explain why such a result could support the often numerical observed existence of homeomorphism “far from the identity” that solves the conjugacy problem in many applications. To conclude, connections between this work and the averaging of time-periodic systems without a small parameter, developed by Chekroun *et al.* (DCDS Ser. A, Vol. 14 (4), 2006), will be explained. Parts of this talk represent joint work with Michael Ghil, Jean Roux, and Eric Simonnet.

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## Instabilities in higher dimensional Hamiltonian systems

**Rafael De la llave**

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**Amadeu Delshams and Tere M.Seara**

We discuss a geometric mechanism that establishes instability in systems with many degrees of freedom. The main idea is that higher order resonances can be countoured while first order resonances can be jumped using previously established mechanisms.

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## Arnold diffusion along nearly parabolic orbits for the planar restricted planar 3-body problem

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**Vadim Kaloshin and Tere M. Seara**

We consider the (elliptic) restricted planar three body problem (RP3BP), with the primaries moving in a elliptic orbit of small but fixed eccentricity about their center of mass. Using geometrical methods, we prove the existence of Arnold diffusion along nearly parabolic orbits.

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## Perturbations of geodesic flows producing unbounded growth of energy

**Marian Gidea**

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**Rafael de la Llave**

We present a simplified proof of a result, obtained by Delshams, de la Llave and Seara, stating that for a generic Riemannian metric on a manifold, a quasi-periodic (time-dependent) perturbation with a generic external potential will produce some orbits whose energy grows unboundedly in time. This is in contrast to time-independent perturbations, where the energy is conserved. This result implies Mather's acceleration theorem and is related to Arnold's diffusion problem.

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## Non-twist KAM Theory

**Alex Haro**

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**Alejandra Gonzalez-Enriquez and Rafael de la Llave**

In this talk we present some rigorous results on the persistence of degenerate tori in families of non-twist symplectomorphisms. The proofs of the results lead to numerical algorithms. We also present some preliminary computations.

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## Existence of Arnold diffusion in a-priori unstable Hamiltonian systems: an example

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**Amadeu Delshams**

In this talk I will show that the geometric mechanism for detecting global instability introduced in a recent paper by Delshams, de la Llave and Seara can be applied to a general class of a-priori unstable Hamiltonian systems. The proof of this results includes a precise study of the effect of the resonances present in the system, which is remarkably useful not only for the problem of diffusion but also for the comprehension of other phenomena present in physical models. I will also consider a particular example and I will show that it admits diffusing orbits that follow this geometric mechanism.

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## Renormalization of flows with Brjuno frequencies

**Hans Koch**

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**S. Kocić**

We describe a renormalization scheme for analytic vector fields  $(\omega, 0)$ -small on  $T^d \times R^m$ , with  $\omega$  a fixed but arbitrary Brjuno vector. It is shown that infinite renormalizability defines a finite codimension manifold and implies the existence of an elliptic invariant torus with frequency vector  $\omega$ . Unlike earlier methods based on renormalization, which we will

sketch for comparison, our approach does not use any continued fractions expansion.

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### Measures of chaotic transport

#### Hector Lomeli

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#### James Meiss

We study smooth families of diffeomorphisms of that have invariant manifolds arising from a perturbation of a saddle connection. In this context, chaotic transport appears naturally. This version of phase space transport is based on structures that are formed of pieces of normally hyperbolic invariant manifolds, called lobes. We present a formula that measures the volume of such lobes and explore their topology and possible measures of their complexity.

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### Computation of rotation numbers and derivatives for invariant curves of general planar maps

#### Alejandro Luque

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#### Jordi Villanueva

Recently, a numerical method to approximate rotation numbers and derivatives in parametric families of circle diffeomorphisms have been developed. The idea of the method is to extrapolate these quantities the averaging of the iterates (or derivatives of the iterates) of an orbit. This approach can be applied to study quasi-periodic invariant curves of planar maps if we are able to construct a circle diffeomorphism carrying the dynamics of the curve. For the case of twist maps, this follows directly from the projection of the iterates on the angular variable, but in general the construction is not so obvious. Our aim is to propose a method to obtain such a circle diffeomorphism for an invariant curve of a general planar map. The construction, that requires analyticity and Diophantine conditions, consists in averaging the iterates of the planar map in order to modify suitably the Fourier coefficients of the curve.

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### Parametrization of invariant manifolds for 3D volume preserving maps

#### Jason Mireles-james

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#### Hector Lomeli

We will describe some semi analytical numerical schemes based on the parameterization method to compute two dimensional invariant manifolds in 3-D volume preserving maps. These calculations, in particular, allow us to compute of the volumes of lobes and several other geometric quantities related to the transport theory of H. E. Lomeli and J. D. Meiss, Chaos 10, 1, 109–121 (2000). Implementation of the numerics for the so called "volume preserving Henon map" will be discussed.

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### Hamiltonian dynamics of magnetofluid models

#### Philip Morrison

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Various kinds of fluid models, generalizations of magnetohydrodynamics, are used to model plasmas in both nature and experimental devices. Because the Eulerian dynamical variables employed are not canonical variables, the systems are Hamiltonian with noncanonical Poisson brackets. These brackets are of the Lie-Poisson form, and are algebraic extensions with the base Lie algebra being the group of canonical transformations of the plane. A particular plasma four-field fluid model that describes collisionless magnetic reconnection is discussed in detail. The Poisson bracket for this system is used to obtain new independent families of invariants, so-called Casimir invariants, three of which are directly related to Lagrangian invariants of the system. The Casimirs are used to obtain a variational principle for equilibrium equations that generalize the Grad-Shafranov equilibrium equation to include flow. Dipole and homogeneous equilibria are constructed. The linear dynamics of the latter is treated in detail in a Hamiltonian context: canonically conjugate variables are obtained; the dispersion relation is analyzed and exact thresholds for spectral stability are obtained; the canonical transformation to normal form is described; an unambiguous definition of negative energy modes (Krein signature) is given; and thresholds sufficient

for energy-Casimir stability are obtained.



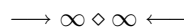
### **On adiabatic perturbation theory for systems with elastic collisions**

**Anatoly Neishtadt**

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**I.V.Gorelyshev**

We consider two problems with elastic collisions: a particle between slowly moving reflecting walls in presence of a slowly varying external potential field, and rays in a smoothly irregular refractive waveguide with reflecting walls. We show that in these problems the standard adiabatic perturbation theory describes dynamics in the mode with collisions with the same accuracy as in the mode without collisions.



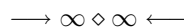
### **Hyperbolicity for symmetric periodic orbits in the three body problem**

**Daniel Offin**

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**Luciano Buono and Kenneth Meyer**

We study subsystems of the N-body problem, constructing minimizing noncollision periodic orbits using a symmetric variational method with a finite order symmetry group. The solution of the variational problem gives existence of such orbits which have a given dihedral subgroup of the full symmetry group in the case of equal masses. The Maslov index of the periodic orbits is then investigated and used to prove the main result which states that the minimizing curves in the reduced energy momentum surface are naturally extended to periodic integral curves which are generically hyperbolic

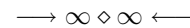


### **Theory and numerics of stability transitions for falling spinning underwater vehicles**

**George W. Patrick**

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Because of the presence of noncompact symmetry, the stability of the motion of a bottom heavy falling, spinning underwater vehicle, in the Kirchhoff approximation, is extremely delicate and complicated. If the vehicle falls too quickly, then this motion is spectrally unstable. As the speed is reduced, there is a (spin dependent) transition to spectral stability, and full nonlinear stability can be demonstrated by KAM confinement. As the speed is further reduced, there is a (spin independent) transition to Energy-momentum stability. In the gap between the transitions, it is expected on general grounds that the addition of arbitrarily small dissipation will induce spectral instability. Since the EM transition is spin independent, and the stability analysis is general for Euclidean symmetry, the implication for gyroscopically stabilized devices is startling: in the presence of noncompact symmetry, robust stability may not be achievable by the use of spin. The Kirchhoff approximation will be reviewed, the stability analysis will be summarized, the numerical validation of the results will be presented.



### **Shadowing orbits for transition chains of invariant tori**

**Clark Robinson**

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**Marian Gidea**

A topological method is used to prove the existence of diffusing orbits for a Hamiltonian system that shadow transition chains of invariant tori connected by either heteroclinic orbits or a Birkhoff zones of instabilities.



### **Biasymptotic trajectories in the Planar and Spatial Restricted 3-Body Problem**

**Pablo Roldán**

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**A. Delshams and R. de la Llave**

For the Restricted Three Body Problem (both the planar and spatial case), we study homoclinic and heteroclinic transport to libration orbits (both Lyapunov and Lissajous) around the equilibrium points  $L_1$  and  $L_2$ . We show how to efficiently encode biasymptotic trajectories using the so-called "scattering map", and how this information can be used to

solve some mission design problems. The same tools were previously applied to near-integrable Hamiltonian models by Delshams, De la Llave, and M.-Seara.

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