

## Special Session 12: Complex and Chaotic Dynamics

Miguel A. F. Sanjuan, Universidad Rey Juan Carlos, Spain  
Jose M. Amigo, Universidad Miguel Hernandez, Spain

### On the topological entropy of the interval maps

**Jose Amigó**

Universidad Miguel Hernandez, Spain  
jm.amigo@umh.es

**Rui Dilão**

Topological entropy is a measure of dynamical complexity. We will talk about different approaches to the topological entropy of interval maps and present some work on progress on its numerical calculation.

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### Transition to chaos in a two-vortex system under oscillatory strain and rotation

**Xavier Carton**

University of Western Brittany, France  
xcarton@univ-brest.fr

**Xavier Perrot**

Vortices are long-lived features of geophysical and laboratory flows. They emerge in free-decay turbulence and govern the long-term evolution of the flow. In 2D flows, two vortices can grow by merger when they are like-signed. The process of vortex merger in 2D flows is well known for a pair of isolated vortices. Nevertheless, in a turbulent field, the velocity field created by neighboring vortices can advect these two vortices away from each other, or exert a strain or shear flow on them. Furthermore, this external velocity field is most often time-varying since these neighboring vortices are not steady. Since vortex merger is strongly dependent on the distance between them, we first investigate the dynamics of a pair of point vortices, embedded in an external shear/strain field which mimics the influence of neighboring vortices. This external flow will first be considered as stationary, and then as time varying. The equilibrium positions of the two vortices in a steady external flow and their stability can easily be computed. We focus on neutral equilibria around which vortices rotate periodically. Then we add an unsteady component to the external strain and rotation, and tune its frequency to resonate with the vortex rotation. With a multiple time-scale expansion, we describe the vortex motion in this case, via an amplitude equation.

We obtain the critical value of unsteady strain field, beyond which the vortex escapes its initial circular trajectory. With Poincaré sections of the dynamical system, we also show that chaos progressively fills the phase plane as this unsteady strain component increases. For finite amplitude of the unsteady strain, the vortex trajectory becomes chaotic and escapes the initial separatrices in the phase plane. This study is then generalized to finite-area vortices. It is shown that a properly oriented external strain is critical in bringing initially distant vortices closer to each other, and therefore facilitates vortex merger. Conclusions are drawn for vortices in 2D turbulence.

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### High energy states on an infinite well

**Maria F Correia**

University of Evora, Portugal  
Email mfac@uevora.pt

**C.C. Ramos and S. Vinagre**

We consider the quantum system of a particle confined on an infinite well. We assume there is some physical interaction in such a way that the system changes its state. The state transition is characterized by a non-linear map  $f$ . The occurrence of several interactions are described by the iterates of the initial state under  $f$ . The obtained iterated states will have increasing energy, and many features that depends on the topological properties of the dynamical system induced by  $f$ . We study in particular the iterated states dependence on the topological entropy of  $f$ .

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### Positive topological entropy of coupled map lattice

**Juan I.g. Guirao**

Universidad Politecnica de Cartagena, Spain  
juan.garcia@upct.es

**M. Lampart**

In this talk we present a lattice dynamical system stated by K. Kaneko in [Phys. Rev. Lett., **65**, 1391-1394, 1990]. We prove that this CML (Coupled Map

Lattice) system has positive topological entropy for zero coupling constant.

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### **Complex and chaotic dynamics in discrete-time delayed Hopfield neural networks**

**Eva Kaslik**

West University of Timisoara, Romania

kaslik@info.uvt.ro

**St. Balint**

There are three main directions in the analysis of neural network dynamics: discovering equilibrium states and periodic or quasi-periodic solutions (of fundamental importance in biological and artificial systems, as they are associated with central pattern generators), establishing stability properties and bifurcations (leading to the discovery of periodic solutions), and identifying chaotic behaviour (with valuable applications to practical problems such as optimization, associative memory and cryptography). In this paper, a bifurcation analysis is presented for a discrete dynamical system with delays describing a Hopfield-type neural network. This analysis allows the description of the stability domain of the null solution and the types of bifurcation occurring at its boundary, in terms of the characteristic parameters. By applying the centre manifold theorem and the normal form theory, all the bifurcations are analyzed. It is shown that the dynamics become more and more complex as the characteristic parameters leave the stability domain, eventually leading to the installation of chaotic behaviour. The route from stability towards chaos passes through several stages of strange attractors and periodic solutions. Numerical results are presented to substantiate the theoretical findings.

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### **Phase control of escapes in open dynamical systems**

**Jesús Seoane**

Universidad Rey Juan Carlos, Móstoles, Spain

jesus.seoane@urjc.es

**S. Zambrano, S. Euzzor, R. Meucci, F. T. Arecchi and M. A. F. Sanjuán**

We present a study of how to avoid escapes in open dynamical systems in the presence of dissipation and forcing, as it occurs in realistic physical situations. We use as a prototype model the Helmholtz oscillator, which is the simplest nonlinear oscillator with escapes. For some parameter values, this oscillator presents a critical value of the forcing for which all particles escape from its single well. By using the phase control technique, weakly changing the shape of the potential via a periodic perturbation of suitable phase  $\phi$ , we avoid the escapes in different regions of the phase space. We provide numerical evidence, heuristic arguments and an experimental implementation in an electronic circuit of this phenomenon. Finally, we expect that this method might be useful for avoiding escapes in more complicated physical situations.

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### **Topological detection of determinism in time series**

**Samuel Zambrano**

Universidad Rey Juan Carlos, Mostoles, Spain

samuel.zambrano@urjc.es

**J. M. Amigó and M. A. F. Sanjuán**

In this work we focus on the detection of determinism in multivariate noisy time series. Our method is topological in the sense that it is based on the properties of topological permutation entropy. It proceeds by counting the number of the so-called ordinal patterns in independent samples of length  $L$  from the data sequence and performing a  $\chi^2$  test, the null hypothesis being that the data are white noise, which implies that all possible ordinal patterns of a given length should be visible and evenly distributed. The dependence of our method on both the presence of observational white noise and the dimension of the underlying attractor is explored numerically in detail.

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