Special Session 13: Asymptotic Behavior of PDEs

Alain Miranville, Universita de Poitiers, France
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Attractors for systems of coupled wave/plate equations with localized damping

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The talk will report on recent progress in the study of the long-term behaviour of some composite systems of Partial Differential Equations (PDE) which arise in the modeling of fluid/structure interactions. Such PDE systems comprise a wave equation in a three dimensional bounded domain and an elastic (or thermoelastic) plate equation acting on a portion of its boundary. Previous work by the author (with Igor Chueshov and Irena Lasiecka) established, in particular, the existence of a global attractor of finite fractal dimension for a coupled wave/plate model with interior nonlinear dissipation and nonlinear perturbations acting on either component of the system. Significant recent advances in the study of wave equations made it possible to show that the aforementioned results can be actually improved, allowing a localized—rather than fully interior—wave damping. It is important to emphasize that the major difficulties come from the combination of nonlinear localized dissipation and (nonlinear) "critical" restoring forces. The main part of the talk will focus on results jointly obtained with Daniel Toundykov (University of Nebraska-Lincoln).

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Robust exponential attractors for singularly perturbed Hodgkin-Huxley equations

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In 1952, A.L. Hodgkin and A.F. Huxley proposed a well-known nonlinear one-dimensional reaction-diffusion system to model the nerve conduction in the giant axon of the squid Loligo. In this system, the parabolic equation which governs the electrical potential in the nerve can be replaced by a more appropriate hyperbolic type equations characterized by a (small) parameter (i.e., the inductance coefficient). The resulting system is a singular perturbation of the original Hodgkin-Huxley equations. The global dynamics of the H-H equations, perturbed or not, is rather complex. This fact justifies a qualitative approach whose first goal is to prove the existence of small geometric sets which describe the nontransient dynamics (typically, global and exponential attractors). In addition, one may wonder how these invariant objects depend on the inductance coeffient when it vanishes. A first "dynamical system" approach was used by W.E. Fitzgibbon, M. Parrott and Y. You in 1996. They established the existence of a global attractor of finite fractal dimension and its upper semicontinuity as the inductance coefficient goes to zero. Then, in 1998, C. Galusinki constructed a family of stable exponential attractors. In this talk, taking advantage of recent progresses in the theory of exponential attractors, we present some refinements of the quoted results. In particular, we show the existence of a family of "robust" exponential attractors in the sense that the symmetric Hausdorff distance between the perturbed exponential attractor and the (lifting of the) unperturbed one can be controlled by a constant time a power of the inductance coefficient.

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On the Caginalp system with dynamic boundary conditions and singular potential

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Stefania Gatti and Alain Miranville

We will consider the Caginalp system of partial differential equations in a bounded smooth domain Ω of R^3:

$$\begin{cases} \varepsilon\,\partial_t w - \Delta w = -\partial_t u, \\ \partial_t u - \Delta u + f(u) = w, \end{cases}$$

This system of equations models melting-solidification phenomena in certain classes of materials. Here, w corresponds to the relative temperature and u is the order parameter, or phase field, which describes the proportion of either of the phases. Moreover we will consider a singular potential, and supplement the equations with Neumann boundary conditions for w and dynamic boundary conditions for u. Then we will prove the existence and uniqueness of solutions, as well as their regularity. Furthermore, when the nonlinear function g is assumed to be positive/negative close to the singularities of the potential, we will prove the existence of global attractors. We will also study the convergence of trajectories to steady states by using an approach based on the so-called Lojasiewicz-Simon inequality.

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Asymptotic behavior of evolutionary systems without uniqueness

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We present an abstract framework to study the asymptotic behavior of a dissipative evolutionary system without uniqueness with respect to weak and strong topologies, which was first introduced primarily to study the long-time behavior of the 3D Navier-Stokes equations. Each evolutionary system possesses a global attractor in the weak topology, but not necessarily in the strong topology. We will discuss the structure of the global attractor for an abstract evolutionary system, and weak/strong uniform tracking properties of omega-limits and global attractors. In addition, we will discuss some applications to nonautonomous systems.

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Bifurcation of the Attractor for Chafee-Infante Equation with Parameter Dependent Boundary Conditions

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In this talk, we consider the Chafee-Infante equation $u_t = u_{xx} + \lambda(u - u^3)$ on $[0, \pi]$ with Dirichlet boundary conditions $u(0) = 0$ and $u(\pi) = \epsilon$. For homogeneous Dirichlet boundary conditions (i.e. $\epsilon = 0$), Chafee-Infante (1974) and Henry (1981) proved the existence a global attractor consisting of equilibria connected by their stable and unstable manifolds, and determined the stability of these equilibria. We will discuss how the structure of the attractor changes as we vary $\epsilon \neq 0$ (i.e. the appearance of new equilibria and their stability).

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Uniform attractors for non-isothermal Cahn-Hilliard equations with dynamic boundary conditions

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We consider a model of non-isothermal phase separation taking place in a confined container. The order parameter ϕ is governed by a viscous or non-viscous Cahn-Hilliard type equation which is coupled with a heat equation for the temperature θ. The former is subject to a nonlinear dynamic boundary condition recently proposed by physicists to account for interactions with the walls, while the latter is endowed with a standard (Dirichlet, Neumann or Robin) boundary condition.

We indicate by α the viscosity coefficient, by ε a (small) relaxation parameter multiplying $\partial_t\theta$ in the heat equation and by δ a small latent heat coefficient (satisfying $\delta \leq \lambda\alpha$, $\delta \leq \overline{\lambda}\varepsilon$, $\lambda, \overline{\lambda} > 0$) multiplying $\Delta\theta$ in the Cahn-Hilliard equation and $\partial_t\phi$ in the heat equation. We analyze issues like well-posedness and the asymptotic behavior of the solutions within the theory of infinite-dimensional dynamical systems (that is, global and exponential attractors and their stability with respect to these parameters). We also intend to present results about convergence to equilibria.

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Stability and instability of equilibria on singular domains

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Lukasz Bolikowski and Nicolas Varchon

We consider the reaction-diffusion equation, with Neumann boundary conditions, defined on a disconnected domain Ω, and we compare it to the problem posed on a sequence of connected domains Ω_n converging to Ω in the sense of Mosco. We are inerested in particular in domains with a 'splitting' inside. We prove invariance of stability properties for the steady states. In case when all the steady states are hyperbolic, we prove that the number of stable (unstable) solutions on Ω is equal to that of stable (unstable) solutions on Ω_n, for n large enough. We also show results of numerical experiments.

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Discrete Nonlinear Schrodinger Equations

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Ezzedine Zahrouni (Monastir)

We consider a semi-discrete in time Crank-Nicolson scheme to discretize a damped forced nonlinear Schrödinger equation. This provides us with a discrete infinite-dimensional dynamical system. We prove the existence of a finite dimensional global attractor for this dynamical system. This is a joint work with E. Zahrouni (Monastir)

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Long time behavior in nonlinear thermoelasticity.

Irena Lasiecka
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I. Chueshov

Asymptotic behavior of solutions to nonlinear thermoelastic plate equations will be discussed. A distinct feature of the wiork is that (i) the nodel under consideration has no mechanical dissipation and (ii) nonlinear therms)internal forces) are of critical exponents,. As such, the nonlineraity enetring the model lacks compactness with repsect to the phase space. Nevertheless we will show that the ultimate-long time behavior of the corresponding evolutions is described by finite dimensional and smooth attractor. Existence of exponetial attractors will be discussed as well.

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Two-dimensional reaction-diffusion equations with memory

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M. Conti, S. Gatti and M. Grasselli

In a two-dimensional space domain, we consider a reaction-diffusion equation whose diffusion term is a time convolution of the Laplace operator against a nonincreasing summable memory kernel k. This equation models several phenomena arising from many different areas. After rescaling k by a relaxation time $\epsilon > 0$, we formulate a Cauchy-Dirichlet problem, which is rigorously translated into a similar problem for a semilinear hyperbolic integro-differential equation with nonlinear damping, for a particular choice of the initial data. Using the past history approach, we show that the hyperbolic equation generates a dynamical system, which is dissipative provided that ϵ is small enough, namely, when the equation is sufficiently "close" to the standard reaction-diffusion equation formally obtained by replacing k with the Dirac mass at 0. Then, we provide an estimate of the difference between ϵ-trajectories and 0-trajectories, and we construct a family of regular exponential attractors which is robust with respect to the singular limit $\epsilon \to 0$. In particular, this yields the existence of a regular global attractor of finite fractal dimension. Convergence to equilibria is also examined. Finally, all the results are reinterpreted within the original framework.

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Numerical study of the asymptotic behaviour of a viscous diffusion equation

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We present a space semidiscretization of the pseudoparabolic equation

$$(PP) \quad u_t - \nu\Delta u_t = \Delta f(u) \quad x \in \Omega,\ t > 0,$$

completed with Neumann boundary conditions and an initial condition, where $\nu > 0$ and $f(s) = s^3 - s$. Seeing (PP) as a nonlocal ODE in $L^\infty(\Omega)$, Pego and Novick-Cohen (1991) proved that a smooth solution may achieve a discontinuous asymptotic state. In order to allow discontinuities, we propose a spatial mixed finite element semidiscretization of (PP) where the approximation u^h of u is a piecewise constant function. We show that our semidiscretization is a gradient flow on an adequate finite dimensional affine space, thus reproducing the dynamics of the continuous case. We also prove uniform convergence on finite time intervals, in space dimension $d \leq 3$. Numerical simulations illustrate the theoretical results. Our approach shows that, starting with values in $[-1, 1]$, u does not in general remain in the physically relevant interval $[-1, 1]$.

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On the dimension of the attractor for the wave equation with nonlinear damping

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We consider a wave equation with nonlinear damping in a bounded domain in R^d. Under certain restrictions on the growth of nonlinearities (in particular, the problem is subcritical) we estimate the dimension of the global attractor. The results are reasonable since we have polynomial (not exponential) dependence on the data. Our estimates are also comparable to one's derived in a (much simpler) case of linear damping, where a a different method can be used.

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Singularly perturbed damped wave equations in unbounded domains

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For an arbitrary unbounded domain $\Omega \subset \mathbb{R}^3$ and for $\epsilon > 0$, we consider the damped hyperbolic equations

$$(H_\epsilon)\ \ \epsilon u_{tt} + u_t + \beta(x)u - \sum_{ij}(a_{ij}(x)u_{x_j})_{x_i} = f(x,u),$$

with Dirichlet boundary condition on $\partial\Omega$, and their singular limit as $\epsilon \to 0$. Under suitable assumptions, (H_ϵ) possesses a compact global attractor $CalA_\epsilon$ in $H^1_0(\Omega) \times L^2(\Omega)$, while the limiting parabolic equation possesses a compact global attractor $\widetilde{CalA}_0$ in $H^1_0(\Omega)$, which can be embedded into a compact set $CalA_0 \subset H^1_0(\Omega) \times L^2(\Omega)$. We show that, as $\epsilon \to 0$, the family $(CalA_\epsilon)_{\epsilon\in[0,\infty[}$ is upper semicontinuous with respect to the topology of $H^1_0(\Omega) \times H^{-1}(\Omega)$. We thus extend a well known result by Hale and Raugel, letting Ω be unbounded and making no smoothness assumptions on $\partial\Omega$, $\beta(\cdot)$, $a_{ij}(\cdot)$ and $f(\cdot, u)$.

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Embedding finite-dimensional attractors into finite-dimensional spaces

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Eleonora Pinto de Moura

This talk with discuss aspects of embedding finite-dimensional sets into finite-dimensional Euclidean spaces. We introduce the 'Lipschitz deviation', which links abstract embedding results with the theory of approximate inertial manifolds, and show that the attractors of many equations have embeddings whose inverse is H"older continuous with an exponent that can be made arbitrarily close to one by taking the embedding dimension sufficiently large. We also discuss the extension of these results to subsets of Banach spaces.

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Asymptotic Behaviour For a Doubly Nonlinear Allen-Cahn Equation

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This talk will focus on the convergence to steady state for the solutions to degenerate Allen-Cahn equations. The well-posedness of the problem and the existence of exponential attractors will also be discussed.

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Asymptotic behavior of some singular phase transition systems

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Elisabetta Rocca

In this talk we will present some results, partly obtained in collaboration with E. Rocca, concerning existence of global and exponential attractors for some models of phase transition characterized by the occurrence of singular terms. We will mainly focus our attention on some variants of the Penrose-Fife model, which will be investigated for various relevant choices of the heat flux law and of the boundary conditions. In particular, we will give some insight on the techniques which can be used to overcome the lack of coercivity characterizing the heat balance equation.

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On the Hyperbolic relaxation of the Cahn Hilliard equation in 3-D: wellposedness and long time behavior

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The aim of this talk is to present some recent results regarding the so-called Hyperbolic relaxation of the Cahn Hilliard equation in three space dimensions. The analysis will be focused on the issues of the well posedness (both local and global) and on the long time behavior from the point of view of attractors.

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Coupled Stokes-Darcy system with Beavers-Joseph interface boundary condition

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We investigate the well-posedness and asymptotic behavior of a coupled Stokes-Darcy model with Beavers-Joseph interface boundary conditions. In the time-dependent case, the well-posedness is established via appropriate time discretization of the problem and a novel scaling of the system under isotropic media assumption. Such coupled systems are crucial to the study of subsurface flow problems, in particular, flows in karst aquifers. We also show that the time dependent solution converges to the unique steady state solution if the viscosity is small. Other asymptotic behavior will also be considered.

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Exponential attractors for B-Z reaction model

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We consider the Oregonator model for the B-Z reaction in a three-dimensional bounded domain. We construct exponential attractors by showing the squeezing property for the semigroup determined from the model equations and show their continuous dependence with respect to the control parameter included in the model.

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