Special Session 14: Smooth dynamical systems and ergodic theory

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C^1-Generic Symplectic Diffeomorphisms

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Artur Avila and Amie Wilkinson

We prove that for a C^1-generic symplectic diffeomorphism f of any closed manifold, the Oseledets splitting along almost every orbit is either trivial or partially hyperbolic. In addition, if f is not Anosov then all the exponents in the center bundle vanish. This establishes in full a result announced by Mañé in the ICM 1983.

Using this result (together with other recent technology), we prove that a C^1-generic partially hyperbolic symplectic diffeomorphism is ergodic. This later part is a joint work with A. Avila and A. Wilkinson.

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Smooth conjugacy problem in the neighborhood of de la Llave's example.

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Structural stability asserts that if two Anosov diffeomorphisms are close enough then they are conjugate: $hf = gh$. It's known that the conjugacy h is Holder continuous. There are simple obstructions for h to be smooth. Let p be a periodic point of f, $f^n(p) = p$, $g^n(h(p)) = h(p)$. If h were differentiable, then the differentials $D(f^n)(p)$ and $D(g^n)(h(p))$ would be conjugate by the differential of h, and we say that f and g have same periodic data. Consider hyperbolic automorphism of four torus $L(x, y) = (Ax, By)$, where A and B are automorphisms of two tori. De la Llave constructed special perturbations of L that have the same periodic data as L but only Holder conjugate to L. We show that any diffeomorphism close to L with the same periodic data is C^1-conjugate to one of de la Llave's examples. Also we make an attempt to introduce additional moduli of C^1-conjugacy in the neighborhood of L.

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Applications of the measure of maximal entropy in nonpositive curvature, orbit counting and volume estimates

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This talk shows a new construction of the measure of maximal entropy for geodesic flows in nonpositive curvature, and some applications, including precise asymptoticos of closed orbits and precise asymptotics of volume.

This talk reviews some classical theorems of G. A. Margulis and presents recent results in this area, some by G. Knieper and some by the speaker.

This talk also points out some interesting aspects of nonuniformly hyperbolic systems where methods from the uniform theory no longer work but where crucial results are still true.

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The Central Limit Theorem for uniformly strong mixing measures

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For an ergodic system, the theorem of Shannon-McMillan-Breiman states that for every finite generating partition the exponential decay rate of the measure of cylinder sets equals the metric entropy almost everywhere. In 1962 Ibragimov showed that the distribution of the measure of cylinder sets is log-normally distributed provided the measure is strong mixing and its conditional entropy function is sufficiently well approximable. Carleson (1958) and Chung (1960) generalised the theorem of SMB to infinite partitions (provided the entropy is finite). We show that the measures of cylinder sets are log-normally distributed for uniformly strong mixing systems and infinite partitions and show that the rate of convergence is polynomial. Apart from the mixing property we require that a higher than fourth moment of the information function is finite. Also, unlike previous results by Ibragimov and others which

only apply to finite partitions, here we do not require any regularity of the conditional entropy function. We also obtain the law of the iterated logarithm and the weak invariance principle for the information function.

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Exact Formula of the Derivative of the Potential Function of the SRB Measure

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For a C^r-diffeomorphism f on a smooth compact Riemannian manifold possessing a hyperbolic attractor, the potential function for the SRB measure $-\log J^u f(h_f(x))$, the unstable Jacobian is differentiable in a C^r-neighborhood of f with respect to f. The derivative formula in the direction of δf is given by

$$\delta(\log J^u_L f) = Div^u_\sigma X^u + Div^u_L X^u(f(x)) - Div^u_L X^u(x)$$

where X^u, X^s are the projections of the vector field $\delta f \circ f^{-1}$ onto unstable and stable subbundles, $Div^u_\sigma X^u$ is the divergence of X^u with respect to the volume form induced by the SRB measure σ, $Div^u_L X^u$ is the divergence of X^u with respect to the volume form induced by the Lyapunov metric on the unstable manifold, and $J^u_L f$ is the unstable Jacobian with respect to the Lyapunov metric on the unstable manifold. This result improves Ruelle's formula by giving the precise expression for the coboundary term

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ANALYTICITY OF THE SUSCEPTIBILITY FUNCTION FOR UNIMODAL MARKOVIAN MAPS OF THE INTERVAL

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David Ruelle

The susceptibility function was analyzed for a general unimodal Markovian map of an interval (assuming that the map is real analytic). In this talk, We will give an outline how to use the change of coordinate to study a general Markovian map and then how to use transfer operators to study the susceptibility function. We will also give an explaination why this method can be also use to study more general one-dimensional maps. Eventually, as explained in a Ruelle's paper, application of a theorem of Whitney should prove differentiability of the map from a dynamical system to the density of its invariant measure restricted to a suitable set.

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Density of cocycles with positive Lyapunov exponents in the smooth category

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Consider the class of C^r-smooth $SL(2,R)$ valued cocycles, based on the rotation flow on two torus with irrational rotation number α. We show that in this class, cocycles with positive Lyapunov exponents are dense if α satisfies the following Liouville type condition: $\left|\alpha - \frac{p_n}{q_n}\right| \leq C\exp(-q_n^{r+1+\delta})$, where $C > 0$ and $\delta > 0$ are some constants and $\frac{P_n}{q_n}$ is some sequence of irreducible fractions.

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Rigidity of higher rank abelian cocycles with values in diffeomorphisms groups

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Anatole Katok

Based on joint work with A. Katok. We consider cocycles over certain hyperbolic higher rank abelian actions and show rigidity properties for cocycles with values in a Lie group or a diffeomorphism group, which are close to identity on a set of generators, and are sufficiently smooth. The actions we consider are Cartan actions on compact quotients of SL(n,R) or SL(n,C), for n greater than 3. The results rely on a technique developed recently by D. Damjanovic and A. Katok.

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Extreme value distributions for non-uniformly hyperbolic dynamical systems

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and Matthew Nicol (University of Houston)

For an observation $\varphi : X \to \mathbb{R}$ over a (discrete or continuous) measure preserving dynamical system $f_t : X \to X$, consider the maxima along trajectories,

$$M_t(x) := \max_{0 \le s \le t} \varphi(f_s x).$$

One is interested in the existence of constants a_t, b_t such that $a_t(M_t - b_t)$ has a nondegenerate limit in distribution. The iid case is well understood: there are only three types of possible limits, and the limit is determined by the distribution of φ.

P. Collet (2001) proved that for a discrete system modeled by a Young Tower with exponential tail and $\varphi(x) := -\log \operatorname{dist}(x, x_0)$, the extreme value distribution exists for almost each x_0, and the limit is the same as if the observations were iid.

We extend this to certain observations with multiple maxima over more general systems and flows. We achieve this by proving a lifting theorem, and refining the technique of Collet.

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Continued fractions, natural extension maps and their geometric structure

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We describe new types of continued fractions and geometric realizations of their natural extension maps. The domain of such a two-dimensional extension map coincides with a canonically associated attracting set. We analyze the geometric structure of these attracting sets and derive some ergodic properties of the continued fraction maps. This is work in progress with S. Katok.

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Hausdorff dimension of typical and non-typical orbits in one-dimensional holomorphic dynamics

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In this talk we discuss dimension-theoretical properties of rational maps on the Riemann sphere. In particular, we study existence and uniqueness of generalized physical measures for several classes of maps including hyperbolic, parabolic, non-recurrent and Topological Collet-Eckmann maps. These measures have the property that their typical points have maximal Hausdorff dimension. On the other hand, we discuss the size of the set of divergence points (the set of points which are non-typical for any invariant measure). Finally, we show that the dimension $d(f)$ is a continuous and plurisubharmonic function of f on an open and dense subset of Rat_d.

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Lifting measures to inducing schemes

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Yakov Pesin and Samuel Senti

We study the liftability property for inducing schemes considered by Pesin and Senti. We show that under some natural assumptions on the inducing schemes any invariant ergodic Borel probability measure of sufficiently large entropy can be lifted to the tower associated with the inducing scheme. The argument uses the construction of connected Markov extensions due to Buzzi, his results on the liftability of measures of large entropy, and a generalization of some results by Bruin [Bru95] on relations between inducing schemes and Markov extensions.

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