Special Session 15: Topological Methods for Boundary Value Problems

Kunquan Lan, Canada
John R. Graef, University of Tennessee at Chattanooga, USA

Existence of noncontinuable solutions of a system of differential equations

Miroslav Bartusek
Masaryk University, Brno, Czech Republic
bartusek@math.muni.cz

In the lecture a system of nonlinear differential equations

$$y_i' = f_i(t, y_1, \ldots, y_{n-1}) g_i(y_n), \quad i = 1, \ldots, n$$

is investigated. Sufficient (necessary) conditions for the existence of a solution of the above given system with boundary value conditions $\lim_{t \to \tau_-} y_i(t) = C_i$, $i = 1, 2, \ldots, n-1$, $\lim_{t \to \tau_-} |y(t)| = \infty$; $\tau < \infty$ and $C_i \in \mathbb{R}$.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Singular Differential Operators and the Asymptotic Behavior of Eigenvalues of Hankel Integral Operators

John Baxley
Wake Forest University, USA
baxley@wfu.edu
and Grey M. Ballard

For ν a positive constant, let τ be the formal differential operator defined by $\tau u = -x^{-2\nu}(x^{2\nu}u')'$. We show how certain self-adjoint operators in the Hilbert space $L^2(0,1;x^{2\nu})$ corresponding to iterates τ^n of this formal operator hold the key to describing the asymptotic behavior of the eigenvalues of some particular Hankel integral operators acting on the Hilbert space $L^2(0,A;x^{2\nu})$ as $A \to \infty$.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Nonlinear Discrete Fractional Difference Equations

Paul Eloe
University of Dayton, USA
Paul.Eloe@notes.udayton.edu
Ferhan M. Atici

The authors develop sufficently some basic properties of discrete fractional calculus so that linear operators of order ν, $0 < \nu$, can be well-defined. The authors then focus on the case $0 < \nu < 2$ and consider two point conjugate (Dirichlet) type boundary value problems for a nonlinear equation of the form, $Lu = Nu$, where L is a well-defined linear operator and N denotes the nonlinear operator. The authors construct the equivalent fixed point operator for two different types of operators L.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Positive Solutions to a Fourth Order Three Point Boundary Value Problem

John Graef
University of Tennessee at Chattanooga, USA
john-graef@utc.edu
Johnny Henderson and Bo Yang

The authors consider a three-point boundary value problem for the beam equation. Some *a priori* estimates to the positive solutions of the problem are obtained, and sufficient conditions for the existence and nonexistence of positive solutions are established. The results are illustrated with an example.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Three Functionals Fixed Point Theorem

Johnny Henderson
Baylor University, USA
Johnny_Hendersonbaylor.edu
Richard Avery and Donal O'Regan

We present a generalization of the multiple fixed point theorems involving compression and expansion arguments. As an application, the existence of multiple positive solutions for a second order conjugate boundary value problem is considered, and we conclude with an extension to multivalued maps.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Positive solutions of nonlocal boundary value problems with singularities

Gennaro Infante
University of Calabria, Italy

g.infante@unical.it

We discuss the existence of positive solutions for some nonlocal boundary value problem where the boundary conditions involve linear functionals in the space $C[0,1]$ and the involved nonlinearity might be singular. Our main ingredient is the theory of fixed point index.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Positive solutions of a nonlinear fractional order boundary value

Eric Kaufmann
University of Arkansas at Little Rock, USA
erkaufmann@ualr.edu
Nickolai Kosmatov

In this paper we give sufficient conditions for the existence of multiple positive solutions to the nonlinear fractional boundary value problem. The differential operator is the Riemann-Liouville fractional derivative of order α.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Existence of positive solutions for higher order boundary value problems with nonhomogeous boundary conditions

Lingju Kong
University of Tennessee at Chattanooga, USA
Lingju-Kong@utc.edu
Qingkai Kong (kong@math.niu.edu)

We study a $2n$th order nonlinear boundary value problem with $2n$ paremeters $\lambda_0, \ldots, \lambda_{2n-1}$ in the boundary condition. Sufficient conditions are obtained for the problem to have at least one positive solution and to have no solution, respectively. Moreover, under certain conditions, we prove that there exists a bounded and continuous surface Γ separating $R_+^{2n} \setminus \{0, \ldots, 0\}$ into two disjoint subsets Λ^E and Λ^N with $\Gamma \subseteq \Lambda^E$ such that the problem has at least two positive solutions for each $(\lambda_0, \ldots, \lambda_{2n-1}) \in \Lambda^E \setminus \Gamma$, one positive solution for each $(\lambda_0, \ldots, \lambda_{2n-1}) \in \Gamma$, and no positive solution for any $(\lambda_0, \ldots, \lambda_{2n-1}) \in \Lambda^N$.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Nodal solutions of multi-point Boundary Value Problems

Qingkai Kong
Northern Illinois University, USA
kong@math.niu.edu
Lingju Kong and James S. W. Wong

We study the nonlinear boundary value problem consisting of the equation $y^{pp} + w(t)f(y) = 0$ on $[a, b]$ and a multi-point boundary condition. By relating it to the eigenvalues of a linear Sturm-Liouville problem with a two-point separated boundary condition, we obtain results on the existence and nonexistence of nodal solutions of this problem. We also discuss the changes of the existence of different types of nodal solutions as the problem changes.

$\longrightarrow \infty \diamond \infty \longleftarrow$

A multi-point boundary value problems of distributed order

Nickolai Kosmatov
University of Arkansas at Little Rock, USA
nxkosmatov@ualr.edu

We study a differential equation of distributed (fractional) order satisfying multi-point boundary conditions. The differential operator is the Riemann-Liouville fractional derivative.The existence results are obtained via the Schauder fixed point theorem.

$\longrightarrow \infty \diamond \infty \longleftarrow$

On the solvability of some beam equations with functional boundary conditions

Feliz Minhós
University of Évora., Portugal
fminhos@uevora.pt
João Fialho

This work deals with the fourth order problem composed by the nonlinear fully equation

$$u^{(iv)}(x) = f(x, u(x), u'(x), u''(x), u'''(x))$$

where $x \in [0,1]$, $f : [0,1] \times \mathbb{R}^4 \to \mathbb{R}$ is a continuous function, and the functional boundary conditions

$$\begin{array}{l} L_0\left(u, u', u'', u(a), u'(a), u''(a)\right) = 0 \\ L_1\left(u, u', u'', u(a), u'(a), u''(a)\right) = 0 \\ L_2\left(u, u', u'', u(a), u'(a), u''(a), u'''(a)\right) = 0 \\ L_3\left(u, u', u'', u(b), u'(b), u''(b), u'''(b)\right) = 0 \end{array}$$

where L_i, $i = 0, 1, 2, 3$, are continuous functions satisfying some monotonicity assumptions.

Some particular cases of problem (**??**)-(**??**), such as nonlocal and multipoint problems, will be considered.

An application to a continuous model to study the deformation of the human spine with conditions in the interior points will be presented.

The arguments make use of topological tools such as lower and upper solutions technique and degree theory.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Existence and Multiplicity of Solutions for Stagnation Point Flow Toward a Stretching Sheet

Joseph Paullet
Penn State Erie, USA
jep7@psu.edu
Patrick Weidman

This talk will investigate a nonlinear third order ODE boundary value problem derived from stagnation point flow of a fluid toward a stretching sheet. The problem will involve a physical parameter, b, representing the ratio of the stagnation flow strain rate to the stretch rate of the sheet. Using a topological shooting argument, we prove that for all $b > 0$ there exists a monotonic solution to the BVP and that if $b > 1$, this solution is unique.

$\longrightarrow \infty \diamond \infty \longleftarrow$

A periodic boundary value problem

Haiyan Wang
Arizona State University, USA
wangh@asu.edu

We will discuss the existence and multiplicity of positive periodic solutions for a class of nonlinear systems of differential equations. Our methods utilize the Krasnselskii fixed point theorem in cones and superlinear and sublinear assumptions.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Existence of Solutions to Singular Integral Equations

Patricia j. y. Wong
Nanyang Technological University, Singapore
ejywong@ntu.edu.sg

We consider the system of integral equations

$$u_i(t) = \int_0^T g_i(t,s)[a_i(s,u_1(s),u_2(s),\cdots,u_n(s))$$
$$+b_i(s,u_1(s),u_2(s),\cdots,u_n(s))]ds,$$

with $t \in [0,T]$, $1 \le i \le n$ where $T > 0$ is fixed and the nonlinearities $a_i(t,u_1,u_2,\cdots,u_n)$ can be *singular* at $t = 0$ and $u_j = 0$ where $j \in \{1,2,\cdots,n\}$. Criteria are established for the existence of *fixed-sign solutions* $(u_1^*,u_2^*,\cdots,u_n^*)$ to the above system, i.e., $\theta_i u_i^*(t) \ge 0$ for $t \in [0,1]$ and $1 \le i \le n$, where $\theta_i \in \{1,-1\}$ is fixed. We also include an example to illustrate the usefulness of the results obtained.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Positive solutions of a nonlinear higher order three point boundary value problem

Bo Yang
Kennesaw State University, USA
byang@kennesaw.edu
John R. Graef and Lingju Kong

The authors study a higher order three point boundary value problem. Estimates for positive solutions are given. These estimates improve some recent results in the literature. Sufficient conditions for the existence and nonexistence of positive solutions of the problem are also obtained.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Positive solutions of second-order multipoint boundary value problems

Miroslawa Zima
University of Rzeszow, Poland
mzima@univ.rzeszow.pl

We shall discuss sufficient conditions for the existence of positive solutions of the equation

$$-u'' + \omega^2 u = h(t,u), \quad t \in [0,1]$$

subject to multipoint boundary conditions. Our approach is based on the existence results for the perturbed Hammerstein equation due to J.L.R. Webb and G. Infante.

$\longrightarrow \infty \diamond \infty \longleftarrow$