# Special Session 17: Nonlinear Evolution Equations and Related Topics

**Nobuyuki Kenmochi**, Chiba University, Japan
**Toyohiko Aiki**, Gifu University, Japan
**Mitsuharu Otani**, Department of Applied Physics, School of Science and Engineering, Waseda University, Japan

## Doubly nonlinear evolution equations in reflexive Banach spaces

**Goro Akagi**
Shibaura Instittue of Technology, Japan
akagi@aoni.waseda.jp

Let $V$ and $V^*$ be a reflexive Banach space and its dual space, respectively, and let $H$ be a Hilbert space such that $V \hookrightarrow H \equiv H^* \hookrightarrow V^*$. Moreover, let $\varphi$ and $\psi$ be proper lower semicontinuous convex functionals defined on $V$ and $H$ respectively. This talk is concerned with the following abstract Cauchy problem:

$$\begin{aligned} \frac{dv}{dt}(t) + \partial_V \varphi(u(t)) &= \alpha v(t) + f(t), \\ &\qquad v(t) \in \partial_H \psi(u(t)), \\ &\qquad 0 < t < T, \\ v(0) &= v_0, \end{aligned}$$

where $T > 0$, $\alpha \in \mathbb{R}$, $\partial_V \varphi : V \to V^*$ and $\partial_H \psi : H \to H^*$ stand for the subdifferential operators of $\varphi$ and $\psi$, respectively, and $f : (0,T) \to V^*$ and $v_0 \in H$ are given. Moreover, the Cauchy-Neumann problem for the porous medium equation is also treated as an application of the preceding abstract problem.

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## Bifurcation Theory of Both Electron and Hole in AlAs/ GaAs Semiconductor Superlattices.

**Ghader Darbandi**
Material and Energy Research Center, Iran
St_gh.darbandi@mail.urmia.ac.ir

We present Bifurcation analysis of electron and hole in semiconductor superlattices by using the resonance tunneling model. We analyze the dynamics of current in relation to both the electron and the hole in weakly coupled semiconductor superlattices. we hypothesis a simple model was composed of equations of nonlinear transport of both the electrons and the holes from well to well and, tried to understand the dynamical features with the help of Poincare maps with fewer degrees of freedom. We studied out treatment of the holes on bifurcation levels. Our results provide the first evidence of charge dynamic equations both the electrons and the holes in GaAs/AlAs semiconductor superlattices in comparison with the case only consider electron transport in conduction band of system.

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## $H^2$-solutions for some elliptic equations with nonlinear boundary conditions

**Junichi Harada**
Waseda University, Japan
t-tsuku@suou.waseda.jp

In this talk, we consider the existence of $H^2$-solutions for elliptic equations $-\Delta u + bu = f(x)$ under the nonlinear boundary condition $-\frac{\partial u}{\partial n} = \beta(u)$. These nonlinear flux conditions on the boundary often comes from the so-called Stefan-Boltzmann's radiation law, which says that the heat energy radiation from the surface of the body $J$ is given by $J = \sigma(T^4 - T_s^4)$, where $\sigma > 0$ is a physical constant, $T$ is the surface temperature and $T_s$ is outside temperature. This nonlinear flux condition from Stefan-Boltzmann's law implies that $\beta(u)$ is monotone increasing function. In this case, the solvability in $H^2$-space and the uniqueness for the equation is completely covered by the abstract theory by H.Brézis.
owever, if we consider the case where the heat flux radiated from the surface is reflected by its surrounding materials, then we must consider also the absorption effect. For such a case $\beta(u)$ could not be a monotone increasing function.
In fact, such a kind of non-monotone radiation-absorption model are already proposed from the view point of engineering.
In this talk, we are concerned with such a non-monotone radiation-absorption model. Our basic tools to analyse this problem are the variational method and some approximation techniques.

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## Nonexistence of weak solutions of quasilinear elliptic equations with variable coefficients

**Takahiro Hashimoto**
Meteorological College, Japan
thashimoto@mc-jma.go.jp

In this talk, we are concerned with the following quasilinear elliptic equations:

$$(\mathrm{E})\begin{cases} -\mathrm{div}\left\{a(x)|\nabla u|^{p-2}\nabla u\right\} &= b(x)|u|^{q-2}u \quad \text{in } \Omega, \\ u(x) &= 0 \quad \text{on } \partial\Omega, \end{cases}$$

where $\Omega$ is a domain in $\mathbf{R}^N$ $(N \geq 1)$ with smooth boundary.

When $a$ and $b$ are positive constants, there are many results on the nonexistence of nontrivial solutions for the equation (E). The main purpose of this talk is to discuss the nonexistence results for (E) with a class of weak solutions under some assumptions on $a$ and $b$.

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**On a non-isothermal phase separation model with constraints**

**Kouta Kumazaki**
Nagoya Instisute of technolopy, Japan
kouta@cotton.ocn.ne.jp
**and Masahiro Kubo**

We consider non-isothermal phase separation models of the Penrose-Fife type.We impose a non-homogeneous Dirichlet boundary condition to nonlinear heat flux.Morever, we consider Signorini boundary condition for the conserved order parameter.We show the existence and uniqueness of solutions to these models.

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**On a solution with transition layers for a bistable reaction-diffusion equation with spatially heterogeneous enviromnents.**

**Hiroshi Matsuzawa**
Numazu National College of Technology, Japan
hmatsu@numazu-ct.ac.jp

In this talk, we report recent studies on the bistable reaction-diffusion equations with spatially heterogeneous environment on a bounded domain $\Omega$. These equation give rise to sharp transition layers when the diffusion coefficient is very small. It is well known that the configulation of these transition layers are affected by the environment factor. In my talk, we consider the case when this environmental factor degenerates on subset of a domain $\Omega$.

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**A fast-reaction slow diffusion limit for propagating redox fronts in mineral rocks**

**Adrian Muntean**
Technische Universiteit Eindhoven, Netherlands
a.muntean@tue.nl

We consider a fast reaction-slow transport problem modeling the flow and molecular diffusion of an aqueous oxidant that reacts with an immobile reductant to produce mineral deposits. We investigate the fast-reaction slow-diffusion limit and indicate that if $\epsilon$ goes to zero, then the limit problem becomes a hyperbolic Stefan-like problem with kinetic condition at the moving interface. We report on preliminary results obtained together with S. Martin (Orsay, Paris 11, France).

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**Elliptic Quasi-Variational Inequalities**

**Yusuke Murase**
Graduate School of Science and Technology, Chiba University, Japan
ym@graduate.chiba-u.jp
**R. Kano, N. Kenmochi**

We consider a class of quasi-variational inequalities of the elliptic type, which arises in composite materials. For instance, the following system of elliptic PDE and variational inequality with unknown dependent convex constraints;

$$\begin{cases} u \in K_0(\theta); \\ -\Delta\theta = h(\theta, u) \quad \text{in } \Omega \ \text{ on } \ \partial\Omega \\ \displaystyle\sum_{i=1}^{l} \int_\Omega a_i(u)\nabla u_i \cdot \nabla(u_i - v_i)dx \\ \qquad \leq \displaystyle\int_\Omega f \cdot (u - v)dx \quad \text{for } \ ^\forall v \in K_0(\theta) \end{cases}$$

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**Global existence and long-time behaviour of Cahn-Hilliard system coupled with viscoelasticity**

**Irena Pawłow**
Systems Research Institute, Polish Academy of Sciences and Military University of Technology, Warsaw, Poland
pawlow@ibspan.waw.pl

A three-dimensional Cahn-Hilliard system coupled with viscoelasticity is considered. The system arises as a model of phase separation process in a binary, deformable alloy.

The existence and uniqueness of a global in time, regular solution is proved. The main ingredient of the proof are estimates of absorbing type with the property of exponentially time-decreasing contribution of the initial data. Such estimates allow not only to prolong the solution step by step on the infinite time interval but also to conclude the existence of an absorbing set.

It is proved that for a sufficiently regular initial data the trajectory of the solution converges to the $\omega$-limit set of these data. Moreover, it is shown that every element of the $\omega$-limit set is a solution of the corresponding stationary problem.

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**Extensions of bifurcation from simple eigenvalue theorem**

**Jun-ping Shi**
College of William and Mary, USA
shij@math.wm.edu
**Ping Liu, Yuwen Wang and Xuefeng Wang**

Nonlinear equations can often be formulated as a nonlinear operator equation in Banach spaces, and the linearized operators are often linear Fredholm operators of index zero. Examples include a large class of quasilinear elliptic systems with nonlinear boundary conditions. A classical theorem of Crandall and Rabinowitz states that a bifurcation occurs at a simple eigenvalue along a branch of trivial solutions, and near the bifurcation point, the solution set consists of a line segment and a curve crossing it. We show a bifurcation theorem generalizing the classical one without a priori knowledge of a line of trivial solutions, which can also be used in perturbation problem of degenerate solutions. In another extension we show that the bifurcation from a simple eigenvalue is indeed global, which is also an extension of Rabinowitz global bifurcation theorem which requires compactness.

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**Two positive solutions for an inhomogeneous scalar field equation**

**Tatsuya Watanabe**
Waseda University, Japan
tatsuw@kurenai.waseda.jp

We consider the following equation:

$$-\Delta u + u = g(u) + f(x), \ \ x \in \mathbb{R}, \qquad (1)$$

where $N \geq 3$. When $f(x) \equiv 0$, it is known that there is a nontrivial solution of (1) for wide classes of nonlinearities $g$. Even though $f(x) \not\equiv 0$, we can expect the existence of a nontrivial solution if $f(x)$ is small in a suitable sense. Our purpose is to show the existence of positive solutions of (1) via the variational approach when $\|f\|_{L^2}$ is small. Compared with assumptions in the case $f(x) \equiv 0$, we only require stronger assumptions on the behavior of the nonlinearity $g$ near zero.

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**Approximating problems of the singular diffusion equations with inhomogeneous terms**

**Noriaki Yamazaki**
Muroran Institute of Technology, Japan
noriaki@mmm.muroran-it.ac.jp
**Hirotoshi KURODA**

Let us consider the singular diffusion equations with inhomogeneous terms in one dimensional case as follows:

$$u_t - \frac{1}{b(x)} \mathrm{div}\left( a(x) \frac{u_x}{|u_x|} \right) = 0 \quad \text{in } (0,L)\times(0,T), \qquad (2)$$

where $L > 0$, $T > 0$ are given constants, and $a(x)$, $b(x)$ are given positive functions. Also, $u : (0,L) \times (0,T) \to \mathbb{R}^N$ is the unknown function, and $u_t$ denotes the time derivative.

In this talk, we consider the approximating problems of (2). Then, we show the existence-uniqueness and the convergence of solutions to our approximating equations. Moreover, we show some examples of stationary solutions, and give the numerical experiments of our problems.

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**A chemotaxis model with general Wentzell boundary condition**

**Zhenbu Zhang**
Jackson State University, USA
zhenbu.zhang@jsums.edu

In this talk, we will consider a chemotaxis model with general Wentzell boundary condition. We will investigate the well-posedness, existence, pattern formation and other properties of the solution.

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