Special Session 18: Topological Dynamics

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Dynamics of Spacing Subshifts

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Spacing subshifts are a class of subshifts of a full shift on two symbols, defined in a superficially similar fashion to S-gap shifts. They were introduced by Lau and Zame in 1973 to give accessible examples of maps that are weakly (topologically) mixing but not strongly mixing, but have attracted little attention since. This talk will discuss the variety of topological dynamical behaviours they exhibit, and some conditions under which they are sofic.

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On combinatorics of Newton maps on real polynomial equations

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We consider some dynamical properties of Newton maps associated to general real polynomial equations.

In particular for cubic, quartic and quintic equations we use topological conjugations and Tschirnhaus transformations to obtain their canonical forms and Milnor-Thurston symbolic dynamics to get bounds of the topological entropy of the associated Newton maps, N_f

We prove also a monotone dependence of the entropy of N_f with respect to parameters of the canonical forms.

Aditionally we get that close to attracting fixed points of Newton maps N_f, the orbits of points follow Bendford's law: the proportion of values in $\{x, N_f(x), ..., N_f^n(x)\}$ with mantissa (base b) less than t tends to $log_b t$ for all $t \in [1, b)$ as $n \to \infty$ for all integer bases $b > 1$.

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When are indecomposable sets omega-limit sets?

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Brian Raines (Baylor) and Robin Knight (Oxford)

Let $f : X \to X$ be a continuous function on a compact metric space. We say that $\Lambda \subseteq X$ is internally indecomposable, if for any $x, y \in X$ and any $\epsilon > 0$, there is an ε-pseudo-orbit in Λ from x to y.

We look at situations (for example symbolic dynamical systems and tent maps) when closed, f-invariant, internally indecomposable sets are ω-limit sets. In particular we relate internal indecomposability and Šarkovskii's property, that a proper nonempty closed subset A of $\omega(x)$ meets $\overline{f(\omega x - A)}$.

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Periodic structure of Morse-Smale diffeomorphims on the torus

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The aim of the talk is, by using Lefschetz zeta function, to study the set of periods of the Morse–Smale diffeomorphisms defined on the two–dimensional torus for every homotopy class.

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Inverse limits and the problem of backward dynamics in economics

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In several standard economics models, the problem of "backward dynamics" arises, i.e., from the model a map arises which is well defined going backward in time, but is not well defined going forward in time. Of course, economists would like to predict the future, and need techniques that can handle this situation. We use inverse limits to overcome the problem, and have obtained both quantitative and qualitative results.

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Invariant Measures on Stationary Bratteli Diagrams

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All ergodic probability measures on the path space of stationary Bratteli diagrams which are invariant with respect to the tail equivalence relation are described. In particular, this description gives a feasible algorithm of finding invariant measures for any aperiodic substitutional system. Several corollaries of this result are obtained.

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Hyperbolicity of C^1 stably expansive homoclinic classes

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Let $f : M- > M$ be a diffeomorphism defined in a d-dimensional compact boundary-less manifold M. We introduce the notion of C^1 stable expansivity for a closed f-invariant set, and study the hyperbolicity of C^1 stably expansive homoclinic classes.

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Topological Computations for Area and Volume Preseving Maps.

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We will discuss some topological and set oriented numerical methods which can be used to study the dynamics of area and volume preserving maps. Examples will be given for the computation of horseshoes in some specific maps.

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Microdynamics

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We describe a new type of the scaling phenomenon. It has some features similar to renormalization and some similar to intermittency. We call it microdynamics. In a one-parameter family of maps in dimension 2, when the parameter goes to 0, the maps converge to the identity. Nevertheless, after a linear rescaling of both space and time, we get maps with attracting invariant closed curves. As the parameter goes to 0, those curves converge in a strong sense to a certain circle.

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Lyapunov function and stochastic search for the global minimum

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Applying classical concepts from the Lyapunov stability theory to a semi-dynamical system defined on the space of probability measures we present a proof of convergence of a general stochastic search for the global minimum.

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Coherent lists and chaotic sets

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Recently it has been noticed that residual subsets of the hyperspace may serve as a tool in the construction of large chaotic sets. This approach extends results of Kuratowski and Mycielski on independence relations in this new context.

The aim of this talk is to survey through known results and present some new applications.

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Conjugacy of Discrete Dynamical Systems in the Neighbourhood of Invariant Manifold in

Banach Space

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In Banach space $\mathbf{X} \times \mathbf{E}$ the dicrete dynamical system

$$\begin{aligned} x(t+1) &= g(x(t)) + G(x(t), p(t)), \\ p(t+1) &= A(x(t))p(t) + \Phi(x(t), p(t)) \end{aligned} \tag{1}$$

is considered. Sufficient conditions under which there is an Lipschitzian invariant manifold $u\colon \mathbf{X} \to \mathbf{E}$ are obtained. Using this result we find sufficient conditions of conjugacy (1) and

$$\begin{aligned} x(t+1) &= g(x(t)) + G(x(t), u(x(t)), \\ p(t+1) &= A(x(t))p(t). \end{aligned} \tag{2}$$

This result allows one to replace the given system by a much simpler one. Relevant results concerning partial decoupling and simplifying of the semidynamical systems are given also.

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On the tip of the tongue

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Michal Misiurewicz

We study the shape of the boundaries of the tongues, and the behavior close to their tips, for the family of double standard maps $f_{a,b}(x) = 2x + a + (b/\pi)\sin(2\pi x) \pmod 1$. It turns out that the shape is fairly regular, mainly due to the real analyticity of the maps.

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C^1-stable shadowing property on chain components

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The notion of pseudo-orbits very often appears in several branches of the modern theory of dynamical systems, and, especially, the pseudo-orbit shadowing property usually plays an important role in the investigation of stability theory and ergodic theory. In the shadowing theory of dynamical systems, chain recurrent sets (resp. chain components) are one of the basic objects in the investigation, and the sets are natural generalization of hyperbolic non-wandering sets (resp. basic sets) in Smale's dynamical systems theory.

Let f be a diffeomorphism of a closed C^∞ manifold. In this talk, we define the notion of the C^1-stable shadowing property for a closed f-invariant set, and prove that (i) the chain recurrent set $\mathcal{R}(f)$ of f has the C^1-stable shadowing property if and only if f satisfies both Axiom A and the no-cycle condition, and (ii) for the chain component $C_f(p)$ of f containing a hyperbolic periodic point p, $C_f(p)$ has the C^1-stable shadowing property if and only if $C_f(p)$ is the hyperbolic homoclinic class of p.

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Cylinder cocycle extensions of minimal rotations on monothetic groups

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Let X be a compact metric space and $T : X \longrightarrow X$ be a homeomorphism of X. Let $\varphi : X \longrightarrow \mathbb{R}$ be a continuous function (called *a cocycle*). By a *cylinder transformation* we mean a homeomorphism $T_\varphi : X \times \mathbb{R} \longrightarrow X \times \mathbb{R}$ (or rather a $\mathbb{Z}$–action generated by it) given by the formula

$$T_\varphi(x, r) = (Tx, \varphi(x) + r).$$

We will also consider the case $\mathbb{R}^m$ instead of $\mathbb{R}$.

As such a transformation cannot be itself minimal (see [1] or [2]) we will focus on the problem of minimal subset. The problem of the minimal subsets of a cylinder transformation turns out to be related to the problem of possible forms of ω-limit sets. H. Poincaré was the first to consider flows (generated by differential equations) on $\mathbb{R}^3$ that had time one homeomorphisms topologically isomorphic to cylinder cocycle extensions over irrational rotations and to investigate vertical cross-sections of their ω-limit sets ([4]).

The main result says that there are no minimal sets for any topologically transitive cylinder transformation defined by bounded variation cocycle over an irrational rotation on the circle and over an odometer. It is also shown that for every irrational rotation on the circle there exists an absolutely continuous cocycle that generates a topologically transitive cylinder transformation. Moreover, the only compact monothetic groups that do not admit topologically transitive cocyles are finite cyclic groups.

All the above results comes from [3].

Also a new geometrical approach to the $\mathbb{R}$-cocycles over odometers will be presented.

References

[1] A.S. Besicovitch, *A problem on topological transformations of the plane. II*, Proc. Cambridge Philos. Soc. **47**, (1951), 38-45.

[2] P. Le Calvez, J.-Ch. Yoccoz, *Un théoreme d'indice pour les homéomorphismes du plan au voisinage d'un point fixe.* (French. English summary) [*An index theorem for homeomorphisms of the plane near a fixed point*], Ann. of Math. (2) 146 (1997), no. 2, 241–293.

[3] M.K. Mentzen, A. Siemaszko, *Cylinder cocycle extensions of minimal rotations on monothetic groups*, Colloq. Math. 101 (2004), NO.1, 75-88.

[4] H. Poincaré, *O krivykh opred'eliayemykh different'ialnymi uravneniami* (Russian) [On curves defined by differential equations], Moskva, Ogiz, 1947.

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Semiconjugacy and distributional chaos

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Piotr Oprocha

We provide a sufficient condition for distributional chaos (DC1) to transfer via semiconjugacy.

Let X, Y be compact metric spaces, $f \in C(X)$, $g \in C(Y)$ and $h : X \longrightarrow Y$ be a semiconjugacy. It was proved in [P. Oprocha and P. Wilczyński. *Distributional chaos via semiconjugacy. Nonlinearity*, 20(11):2661–2679, 2007.] that if $(Y, g) = (\Sigma_2, \sigma)$ and $\#h^{-1}(\{p\}) = 1$ for some periodic point $p \in Y$, then (X, f) is uniform distributionally chaotic.

We present a strenghtened version of the statement. Now (Y, g) is allowed to have only WSP and $\#h^{-1}(\{p\}) \leq 2$ for some periodic point $p \in Y$.

We also provide an example of a nonautonomous ordinary differential equation on the plane of the form (in complex number notation)

$$\dot{z} = \overline{z}^2 \left(1 + e^{i\kappa t} \mid z \mid^2\right) + c$$

for which the Poincaré operator on some compact subset of $\mathbb{C}$ is semiconjugated by Φ to a sofic shift and $\#\Phi^{-1}(\{0^\infty\}) = 2$.

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