Special Session 19: Multiscale numerical methods for partial differential equations

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Stochastic multiscale finite element methods and their applications to uncertainty quantification

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In this talk, I will discuss the stochastic multiscale finite element methods. The goal is to construct basis functions which can capture the spatial heterogeneities as well as the uncertainties. We discuss such methods and demonstrate numerical examples for highly heterogeneous porous media. Further, I will discuss how these methods can be used to speed-up the sampling of the subsurface properties.

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Sparse finite element method for multi-scale problems

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We discuss a finite element method for the limiting equations obtained by the multiscale convergence method for multiscale elliptic problems. The limiting problems are posed in high dimensional tensorised domains: if the original problems are posed in $\mathbb{R}^d$ and depend on n scales, the limiting problems depend on n variables in $\mathbb{R}^d$, one for each scale. The problems are descretized by a sparse finite element method, which requires essentially the same number of degrees of freedom and achieved essentially the same rate of convergence as for a conventional finite element method for a partial differential equation in $\mathbb{R}^d$. We apply the method to both linear and nonlinear monotone multi-scale problems.

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An up-scaling algorithm for non-Darcy flows in inhomogeneous porous media

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The goal of this work is to develop a mathematically rigorous framework to study the dynamical processes associated to nonlinear Forchheimer flows in highly heterogeneous reservoir. Using fundamental geometric methods, we have proved the existence of a nonlinear scaling operator which relates constant mean curvature (CMC) surfaces and time invariant pressure distribution graphs constrained by the Darcy-Forchheimer law. This trade between hydrodynamic and geometrical problems can be used as a homogenization method for the numerical solution of the hydrodynamical problem.

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Global mixed multiscale finite element methods and their application in porous media

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In this talk, we present a mixed multiscale finite element method using limited global information. We consider a general case where multiple global information is given such that the solution smoothly depends on these global fields. The global fields typically contain small scale (local or global) information required for achieving a convergence with respect to the coarse mesh size. We present a rigorous analysis and show that the proposed mixed multiscale finite element methods converge. Some preliminary numerical results are shown. We study a parameter dependent permeability field. Using a few global fields corresponding to realizations of permeability fields, we show that one can achieve high accuracy in numerical simulations.

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Adaptive Multi-Scale Algorithm for Multi-Phase Transport in Porous Media

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In the multiscale finite volume (MSFV) method of Jenny et al. (JCP 217: 627–641, 2006) an efficient, accurate coarse scale operator was proposed for pressure in the elliptic flow equation; and a domain decomposition of Schwarz overlap was employed to solve the hyperbolic (or parabolic) transport equation for saturations in fine scale. A coarse scale operator for the transport equations has not been derived because of the hyperbolic characteristics of the governing equations and intricate nonlinear interactions of saturation front and underlying heterogeneous permeability distribution. In this paper we propose a numerical, adaptive coarse scale operator for the transport equation of saturation that will greatly improve numerical efficiency over the previous MSFV. A fully implicit multiscale finite volume algorithm for multi-phase flow in porous media is mathematically framed with prolongation and restriction operations as in a multigrid method. The sequential fully implicit algorithm of MSFV is also applied, in which pressure is computed first and then the transport equation of saturation is solved in sequence.

Two prolongation operators are derived for the transport equations that can be adaptively utilized in the coarse grids in which velocity and/or saturation changes can be represented by asymptotic linear approximations. The numerical efficiency and accuracy are evaluated for two-phase, incompressible flow in two dimensional porous media. The new adaptive algorithm is tested extensively with various models with homogeneous and heterogeneous permeabilities. It is demonstrated that the multiscale results with the adaptive transport calculation are in excellent agreement with the fine-scale solutions. Furthermore, the adaptivity of flow and transport equations in the MSFV of this paper yields a computationally much more efficient algorithm over conventional finite difference reservoir simulation.

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Efficient numerical techniques for turbulent diffusion transport governed by cellular flows

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In this talk we develop efficient numerical techniques for turbulent diffusion transport governed by cellular flows. We perform numerical implementations for the asymptotic approach to approximate the solutions of steady advection-diffusion problem at high Péclet numbers. Spectral methods and finite difference scheme with exponential grids are used in solving heat equations over the unbounded domain due to the asymptotic problem.

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Multiscale Simulations of Fluid Flows in Deformable Porous Media

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In this work the problem of upscaling fluid flow in deformable porous media is studied. At the microscale, the physics of flow in deformable porous media is described by the fluid-structure interaction (FSI) problem. Currently, the well-established macroscopic models for poroelasticity such as Biot's law can only be applied to linear elastic solids. Furthermore, macroscopic parameters such as average fluid pressure and solid displacements are typically limited to infinitely small deformations at the pore level. In this work numerical upscaling methods based on the stationary FSI problem for deformable nonlinear solid and Stokes flow are developed. The strains in the solid are assumed small but no restrictions are applied on the magnitude of the displacements. The FSI problem is solved numerically by an iterative procedure, which sequentially solves fluid and solid sub-problems. A specific geometry - a long channel with elastic walls - is then considered and an asymptotic solution is derived. The numerical results are shown to coincide with the asymptotics. Furthermore, the asymptotic results is used to obtain an upscaled, Nonlinear Darcy-type equation for the averaged pressure. This result is generalized to 3D and used to develop a Hybrid Multiscale Finite Element Model (HMFEM) which bypasses the explicit homogenization step by building fine-scale information directly into a coarse-scale computational grid. Numerical results are presented and extensions to nonlinear solids are discussed.

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Upscaling Flow through Block Permeability Inclusions by an Analytical Approach

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Flow and transport in natural porous media becomes very challenging when one takes into account the intrinsic heterogeneity features of the geological formations built on the permeability/diffusion coefficient. In this talk, we present an analytical way of obtaining the upscaled coefficient and analytical basis functions to construct the first-order approximation. These apply to heterogeneous coefficients that are periodic and rapidly oscillating and can be defined as step functions describing inclusions in a main matrix. The new contribution involves deriving an analytical approximation for the solution of the periodic cell-problem, obtained by a two-scale asymptotic expansion of the respective heterogeneous equation. The known results in the literature, including the geometric average for the checkerboard geometry, are derived as particular cases and a comparison with some numerical results from the literature is presented. We demonstrate, numerically, the convergence properties of the approximations, by applying it to linear and nonlinear problems of interest in flow in porous media. Further, we apply these results for special cases of transport equations, where we also obtain interesting and useful convergence properties, for increasing surface area of the inclusions. The extension to non-periodic media may also be presented.

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Travel wave solutions to the 2d-Korteweg-de Vries-Burgers equation

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In this talk, we propose a method to study the first integrals of the 2d-Korteweg-de Vries-Burgers equation by applying the theory of commutative algebra. We explore the integrals of motion and obtain a class of travel wave solutions in terms of the Jacobian elliptic functions. Numerical simulations under certain parametric conditions are also illustrated.

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