

## Special Session 20: Partial Differential Equations from Fluid Mechanics and Mathematical Physics

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### Operator Methods for Double-Free-Boundary Fluid Problems in Annular domains

**Andrew Acker**

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In past work, the author has developed the Operator Method to study the existence of convex classical solutions to convex one-layer and multi-layer free-boundary problems in fluid dynamics (see Trans. Amer. Math. Soc. 350(1998), pp. 2981-3020, for example). The general idea was to obtain the convex free boundary either as a convex functional minimizer (convex variational inequalities), or as the limit of a sequence of convex solutions of a corresponding convex fixed point problem for a one-parameter family of convexity-preserving free-boundary-perturbation operators. The purpose of the present paper is to show that the same operator method, based on the same operators, can be generalized to study geometrically-general double-free-boundary problems for annular flows of ideal fluid in two space dimensions, in which the free-boundaries are characterized by a generalized form of Bernoulli's law. This is the first time that the Operator Method has been adapted to an essentially arbitrary geometric context.

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### Exploring 2-D turbulence in the 3-D Rayleigh-Benard system

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**Ciprian Foias and Michael Jolly**

We estimate the dissipation wavenumber, and one related to the enstrophy flux for the 2-D Navier-Stokes equation that results from vertically averaging the 3-D Rayleigh-Benard problem. We also show that the dissipation law holds modulo a factor involving the forcing term, which involves boundary forces and Reynolds' stress.

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### Global Well-posedness of the Three-dimensional Geostrophic Turbulence Mixing Model

**Chongsheng Cao**

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**E.S. Titi**

The three-dimensional viscous Boussinesq equations under hydrostatic balance govern large scale dynamics of atmosphere and oceanic motion. To overcome the turbulence mixing a vertical diffusion is added. We prove the global existence and uniqueness (regularity) of strong solutions to this model in a periodic infinite channel.

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### Remarks on the blow-up problem of the 3D Euler equations

**Dongho Chae**

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We discuss blow-up rates and the blow-up profiles of possible asymptotically self-similar singularities of the 3D Euler equations, where the sense of convergence and self-similarity are considered in various sense. We extend much further, in particular, the previous nonexistence results of self-similar/asymptotically self-similar singularities. Some implications the notions for the 3D Navier-Stokes equations are also deduced. Generalization of the self-similar transforms is also considered, and by appropriate choice of the transform we obtain new a priori estimates for the 3D Euler and the Navier-Stokes equations.

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### Cnoidal wave solutions to Boussinesq systems

**Hongqiu Chen**

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**Min Chen and Nghiem V Ngyyen**

Two different techniques are used to study the cnoidal wave solutions of the Boussinesq systems. First, the existence of periodic travelling-wave solutions for a large family of systems is established by using a topological method. Although this result guarantees the existence of cnoidal wave solutions in a parameter region in the period and phase speed plane, it does not provide the uniqueness nor the non-existence of such solutions in other parameter regions. The explicit solutions are then found by using the Jacobi elliptic function series. Some of these explicit solutions fall in the parameter region where the cnoidal wave solutions are proved to exist, and others do not; so the method with Jacobi elliptic functions provides additional cnoidal wave solutions. In addition, the explicit solutions can be used in many ways, such as in testing numerical code and in testing the stability of these waves.

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**Finite time singularities and global well-posedness for fractal Burgers equation**

**Hongjie Dong**

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**Dapeng Du (Fudan University) and Dong Li (IAS at Princeton)**

Burgers equations with fractional dissipation on  $R * R^+$  or on  $S^1 * R^+$  are studied. With supercritical dissipation, we show that with very generic initial data the equation is locally well-posed and its solution develops gradient blow-up in finite time. In the critical dissipative case, the equation is globally well-posed with arbitrary initial data in  $H^{1/2}$ . Finally, in the subcritical dissipative case, we prove that with initial data in the scaling-invariant Lebesgue space the equation is globally well-posed. Moreover the solution is spatial analytic and has optimal Gevrey regularity in the time variable. These improve recent results by Alibaud, Droniou, Vovelle, and Kiselev, Nazarov, Shterenberg.

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**Closure approximations to micro-macro models of complex fluid**

**Qiang Du**

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**H. Hyon, C. Liu and P. Yu**

The coupling of different scales and physical processes is important in the study of complex fluids. The interaction between the (microscopic) elastic properties and the (macroscopic) fluid motions gives not only the rich and complicated rheological phenomena, but also presents formidable challenges in analysis and numerical simulations. In this talk, we will discuss some recent works on the systematic closure of some micro-macro models of complex fluids.

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**The Two-Dimensional Primitive Equations of the Ocean with an Additive Noise**

**Brian Ewald**

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**Roger Temam and Madalina Petcu**

The primitive equations of the ocean are the Navier-Stokes equations modified by considering them on a thin domain with a rotational Coriolis term. We will add an additive Gaussian noise to them and consider questions of existence and uniqueness of solutions to the resulting stochastic PDE. We will also consider some of the probabilistic properties of the resulting solutions with an eye toward determining whether certain natural questions can be formulated in a way that makes formal sense.

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**Navier Stokes Equation in 3D thin domains under Navier friction and interface boundary conditions**

**Changbing Hu**

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In this talk we study the Navier-Stokes equations with Navier friction boundary and interface boundary condition in 3D thin domains. We prove the global existence of strong solutions to the 3D Navier-Stokes equations when the initial data and external forces are in large sets as the thickness of the domain is small. We generalize the techniques developed to study the 3D Navier Stokes equations in thin domains to the Navier friction boundary and interface boundary condition by introducing a new average operator

in the thin direction according to the spectral decomposition of the Stokes operator  $A_\varepsilon$ . Our analysis relies on the refined investigation of the eigenvalue problem corresponding to the Stokes operator  $A_\varepsilon$  with Navier friction boundary and interface boundary conditions. Part of our presentation is based on the results in [1].

## References

- [1] Navier-Stokes equation in three dimensional thin domain with Navier friction boundary condition, *J. Diff. Equations*, 236(2007), 133-163.

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## Some specific mathematical constraints on 2-D turbulence

**Michael Jolly**

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**R. Dascaliuc and C. Foias**

We derive upper and lower bounds for ensemble averages of energy, enstrophy, and palinstrophy for the 2D periodic Navier-Stokes equations. This is done both in the general case, and in the case where the energy power law for fully developed turbulence holds. In the turbulent case, the bounds are sharp, up to a logarithm, and provide a new lower bound on the Landau-Lifschitz degrees of freedom. We also prove that under the turbulence assumption the estimate of the inertial term via Ladyzhenskaya's inequality is sharp on a significant portion of the global attractor.

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## The Complex KdV Burgers Equation: A study on its series type solutions

**Netra Khanal**

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**Jiahong Wu**

It is known fact that some periodic solutions of complex KdV equation with smooth data blows up in finite time. Wu and Yuan investigated the effect of dissipation on the regularity of solutions of the complex KdV equation. Here, we focus on the series type solution of the complex KdV Burgers equation for the periodic domain and develop the global existence

and uniqueness of classical solution if modes of the initial datum are small enough.

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## Least-Squares Finite Element Methods

**Jaeun Ku**

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Least-squares finite element methods have drawn considerable attention to approximate the solutions of second-order elliptic partial differential equations, elasticity, Stokes and Navier-Stokes equations. In this talk, we will discuss a first-order system least-squares method (FOSLS) based on a first-order system representing a second-order equation. Advantages of least-squares methods over the standard/mixed Galerkin methods will be addressed and various new error estimates of these methods will be presented. In particular, we will provide optimal error estimates with respect to the regularity and approximate spaces when the original problem has low regularity solutions. These estimates can be used to justify the process of using linearization step to solve nonlinear problems. In this regard, possible way of using least-squares solver for Stokes equations to approximate the solutions of Navier-Stokes equations will be briefly discussed.

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## Asymptotic behavior of some doubly nonlinear equations

**Alain Miranville**

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**S. Zelik**

Our aim in this talk is to discuss the asymptotic behavior, in terms of finite dimensional attractors, of doubly nonlinear equations which arise, e.g., in the context of the Allen-Cahn and Cahn-Hilliard theories in phase separation.

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## Asymptotic Behaviour of the Darcy-Boussinesq System at Large Darcy-Prandtl number

**Rana Parshad**

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**Xiaoming Wang**

We study asymptotic behavior of the Global attractor of the Darcy-Boussinesq system. We prove that the global attractor for this system converges to that of the infinite Darcy-Prandtl number model which is a limiting case of the above system. We also show convergence of various statistical properties of the Darcy-Boussinesq system to those of the Infinite Darcy-Prandtl number model.

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### **Estimates for $L^p$ norms for the Kuramoto-Sivashinsky equation**

**Atanas Stefanov**

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**Milena Stanislavova**

For the Kuramoto Sivashinsky equation on a finite interval  $[-L, L]$ , there is an open outstanding conjecture that the (asymptotic)  $L^\infty$  bounds are independent of  $L$ . We will show that estimates of this type can be reduced to finding good (and why not optimal?) solutions to a concrete min-max problem for a Schrödinger operator. We will also put out some speculations about possible solutions of these min-max problems.

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### **On strong solutions of the multi-layer quasi-geostrophic equations of the ocean**

**Theodore Tachim medjo**

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In this talk, we study the multi-layer quasi-geostrophic equations of the ocean. The existence of strong solutions is proved. We also prove the existence of a maximal attractor in  $L^2(\Omega)$  and we derive estimates of its Hausdorff and fractal dimensions in term of the data. Our estimates rely on a new formulation that we introduce for the multi-layer quasi-geostrophic equation of the ocean, which replaces the non-homogeneous boundary conditions

(and the non-local constraint) on the stream-function by a simple homogeneous Dirichlet boundary condition. This work improves previous results.

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### **Computation of Navier-Stokes equations with the pseudostress-velocity formulation**

**Yanqiu Wang**

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We will discuss the pseudostress-velocity formulation for Navier-Stokes equations, its finite element discretization, the decoupling of the discrete mixed system and a multigrid solver for the reduced H(div) type equation. The multigrid solver is proved to have optimal convergence rate for Stokes equations. For Navier-Stokes equations, several numerical experiments in 2D show that the multigrid solver works well for time-dependent problems.

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### **The Kawahara Equation in Weighted Sobolev Spaces**

**Juan-ming Yuan**

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**Netra Khanal and Jiahong Wu**

The initial- and boundary-value problem for the Kawahara equation, a fifth-order KdV type equation, is studied in weighted Sobolev spaces. This functional framework is based on the dual-Petrov-Galerkin algorithm, a numerical method proposed by Shen [?] to solve third and higher odd-order partial differential equations. The theory presented here includes the existence and uniqueness of a local mild solution and of a global strong solution in these weighted spaces. If the  $L^2$ -norm of the initial data is sufficiently small, these solutions decay exponentially in time. Numerical computations are performed to complement the theory.

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