Special Session 21: Nonlinear Evolution Equations and Applications

Vladimir Varlamov, USA
Yue Liu

On the uniqueness of KdV 2-solitons

John Albert
University of Oklahoma, USA
jalbert@ou.edu
Jerry Bona, Nghiem Nguyen

The 2-soliton solutions of the Korteweg-de Vries equation satisfy a fourth-order nonlinear ordinary differential equation which depends on two parameters λ, μ. As is well known, for each fixed choice of (λ, μ) in $\mathbf{R}^2$, the ODE is actually a completely integrable Hamiltonian system with two degrees of freedom.

Here we address the question of whether the soliton and 2-soliton solutions of KdV are the only L^2 solutions of this ODE, as (λ, μ) ranges over $\mathbf{R}^2$. In ruling out alternate solutions, the difficult case seems to be when the eigenvalues of the linearized equation around 0 are two double eigenvalues on the imaginary axis, so there is a four-dimensional center manifold. We discuss the connection between this question and the stability theory of 2-solitons. (Preliminary report.)

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The global weak solutions of the Degasperis-Procesi (DP) equation and its numerical aspects

Baofeng Feng
University of Texas-Pan American, USA
baofeng@gmail.com
Yue Liu,
Department of Mathematics, University of Texas at Arlington, USA

Several nonlinear dispersive partial differential equations, including the Cammassa-Holm (CH) equation and the Degasperis-Procesi (DP) equation, have attracted much attention recently, since they can be viewed as shallow water wave models, and are completely integrable. Moreover, they admit non-smooth solutions such as cuspon, peakon and shock-peakon solutions.

In this talk, we first present the existence of global weak solutions including one- and two-peaked solitons Then, an operator splitting method, which consists of a second-order total variation diminishing (TVD) scheme, and a second-order linearized finite difference scheme, will be proposed for the DP equation. The validity of the scheme is verified by a good agreement between the numerical results and exact solutions.

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Well-posedness and analyticity in the space variable for a higher order periodic mKdV

Heather Hannah
East Central University in Oklahoma, USA
hhannah@ecok.edu

We consider the higher order mKdV equation, so that we are examining those equations with a higher dispersion term of the order m, where m is odd and larger than 3. The corresponding periodic Cauchy problem is in fact well-posed in Sobolev spaces for all $s \geq 1/2$. We then show that the solution to the periodic Cauchy problem for this higher order equation with analytic initial data is analytic in the space variable x at any fixed time t near time zero. However, this analyticity is not guaranteed in the time variable t.

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Dependence of solutions on initial data in Sobolev spaces for the CH equation

Alex Himonas
University of Notre Dame, USA
himonas.1@d.edu
Carlos Kenig and Gerard Misiolek

Using the method of approximate solutions we show that the solution map for the Camassa-Holm (CH) equation is not uniformly continuous in Sobolev spaces with exponent greater than one.

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Nonlinear evolution equations with frctional derivatives on a half-line

Elena Kaikina
University of Mexico, Mexico
ekaikina@matmor.unam.mx

We study the initial-boundary value problem for the initial-boundary value problem on a half-line for the nonlinear evolution equations with a fractional derivative. We study traditionally important problems of a theory of nonlinear partial differential equations, such as global in time existence of solutions to the initial-boundary value problem and the asymptotic behavior of solutions for large time.

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The Semilinear Evolution Equation for Universal Contingent Claims in a General Market Environment

Valery Kholodnyi
Platts Analytics, USA
valery_kholodnyi@platts.com

We present and further develop the concept of a universal contingent claim introduced earlier by the author. This concept provides a unified framework for the analysis of a wide class of financial derivatives. Those, for example, include European, Bermudan and American contingent claims. We show that the value of a universal contingent claim in a general market environment is given by a multiplicative measure also introduced earlier by the author. Roughly speaking, a multiplicative measure is an operator-valued function on a semiring of sets which is multiplicative on the union of disjoint sets. We also show that the value of a universal contingent claim is determined by a, generally speaking, impulsive semilinear evolution equation also introduced earlier by the author.

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Stability and Weak Rotation Limit of Solitary Waves of the Ostrovsky Equation

Steve Levandosky
College of the Holy Cross, USA
spl@mathcs.holycross.edu
Yue Liu

We present two results on solitary waves of the Ostrovsky equation. First, we investigate the behavior of the solitary waves as the rotation parameter goes to zero. We also investigate the properties of the function d(c) which determines the stability of the solitary waves. Using a scaling identity, together with numerical approximations of the solitary waves, we are able to numerically approximate d(c), and thereby determine regions in parameter space where the solitary waves are stable or unstable.

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Chaos in Partial Differential Equations, Navier-Stokes Equations and Turbulence

Y. charles Li
University of Missouri, USA
cli@math.missouri.edu

I will present several new ideas on Chaos in Partial Differential Equations, Navier-Stokes Equations and Turbulence.

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Instability of traveling dispersive waves and water waves

Zhiwu Lin
University of Missouri-Columbia, USA
lin@math.missouri.edu

We consider linear instability of periodic and solitary traveling waves of several classes of dispersive wave models. They include generalizations of KDV, BBM, regularized Boussinesq equations with general dispersive terms, and in particular the 2D full water wave problem. We obtain criteria for the existence of exponentially growing solutions to the linearized problem. The novelty is that we can treat models with nonlocal dispersive terms, for which the spectra problem is out of reach by the traditional Evans function technique. Our method is to reduce the linearized problem to study a family of nonlocal operators, which are also closely related to the bifurcation of traveling waves. For the solitary water waves, our criteria imply the linear instability of large waves which can lead to wave breaking.

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Asymptotics of odd solutions of quadratic nonlinear Schrodinger equations

Pavel Naumkin
University of Mexico, Mexico
pavelni@matmor.unam.mx
Nakao Hayashi

We consider the Cauchy problem for the quadratic derivative nonlinear Schrödinger equation

$$\begin{cases} iu_t + \frac{1}{2}u_{xx} = \partial_x |u|^2, \; x \in \mathbf{R}, \; t > 1, \\ u(1,x) = u_1(x), \; x \in \mathbf{R} \end{cases} \quad (1)$$

in the case of odd initial data $u_1(x)$ from $\mathbf{H}^2(\mathbf{R}) \cap \mathbf{H}^{0,2}(\mathbf{R})$. We prove the global existence in time of solutions to the Cauchy problem and construct the modified asymptotics for large values of time.

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Periodic oscillations in the Gross-Pitaevskii equation with a parabolic potential

Dmitry Pelinovsky
McMaster University, Canada
dmpeli@math.mcmaster.ca

We reformulate the Gross–Pitaevskii equation with a parabolic potential as a discrete dynamical system, by using the basis of Hermite functions. We consider small amplitude stationary solutions with a single zero, called dark solitons, and examine their existence and linear stability. Furthermore, we prove, under appropriate conditions, the persistence of a periodic motion in a neighborhood of such solutions when the parabolic potential is perturbed by a small bounded decaying potential. Our results on existence, stability and nonlinear dynamics of the relevant solutions are corroborated by numerical computations.

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Novikov-Veselov equation and the inverse scattering transform

Samuli Siltanen
Tampere University of Technology, Finland
samuli.siltanen@tut.fi
Matti Lassas and Jennifer Mueller

The Novikov-Veselov (NV) equation is one of the 2+1 dimensional generalizations of the KdV equation. Inverse scattering transform (IST) solution for the non-periodic NV equation has been discussed in the literature [Boiti, Leon, Manna and Pempinelli 1987; Tsai 1994], but only formally. In this talk new rigorous mapping properties are proved for the IST scheme related to the NV equation. There is no smallness assumption on the initial data. These theoretical results extend Nachman's 1996 use of the IST for uniqueness proof for the inverse conductivity problem of Calderón. Furthermore, numerical computations are shown to illustrate the IST evolution. The algorithm is based on the practical medical imaging method (for electrical impedance tomography) developed by the authors together with David Isaacson and Kim Knudsen.

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Well-posedness and weak rotation limit for the Ostrovsky equation

Kotaro Tsugawa
Graduate School of Mathematics, Nagoya University, Japan
tsugawa@math.nagoya-u.ac.jp

We consider the Cauchy problem of the Ostrovsky equation. We first prove the time local well-posedness in the anisotropic Sobolev space $\bar{H}^{s,a}$ with $s > -a/2 - 3/4$ and $0 \leq a \leq -1$ by the Fourier restriction norm method. This result include the time local well-posedness in H^s with $s > -3/4$ for both positive and negative dissipation. We next consider the weak rotation limit. We prove that the solution of the Ostrovsky equation converge to the solution of the KdV equation as $\gamma \to 0$ when the initial data of the KdV equation is in L^2. To show this result, we prove a bilinear estimate which is uniform with respect to γ.

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Riesz potentials of Airy functions and related properties of KdV-type equations

Vladimir Varlamov
University of Texas-Pan American, USA
varlamov@utpa.edu

Riesz potentials (fractional derivatives) are defined as fractional powers of Laplacian. For the 1D case new properties are obtained for Riesz potentials of Airy functions and products of Airy functions. One of such properties is the parallelogram law for the above fractional derivatives. New properties of KdV-type equations are established on the basis of these results.

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A new solution representation for the BBM equation in a quarter plane and the eventual periodicity

Jiahong Wu
Oklahoma State University, USA

jiahong@math.okstate.edu
John M. Hong and Juan-Ming Yuan

We report recent results on the initial- and boundary-value problem for the Benjamin-Bona-Mahony (BBM) equation in a quarter plane. The goal is to understand the periodic behavior (termed as eventual periodicity) of its solutions corresponding to periodic boundary condition or periodic forcing. To this aim, we derive a new formula representing solutions of this initial- and boundary-value problem by inverting the operator $\partial_t + \alpha\partial_x - \gamma\partial_{xxt}$ defined in the space-time quarter plane. The eventual periodicity of the linearized BBM equation with periodic boundary data and forcing term is established by combining this new representation formula and the method of stationary phase.

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