

Special Session 23: Applied Analysis and Partial Differential Equations in Engineering and Sciences

Thomas Hagen, The University of Memphis, USA
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Bifurcation in a System of Functional Equations Modeling Epidemics

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Roberto Castillo and Noel Cavazos

In this work we examine a basic mathematical model describing the spread of a class of infectious diseases. A system of four integral equations represents the SEIR model, where individuals go through stages of being susceptible (S), exposed (E), infective (I), and recovered (R) for constant periods of time intervals. Transcritical bifurcation of steady state solutions can be observed in the system as the contact rate parameter increases. Numerical simulations show that, due to the nonlinearity in the mathematical model, the integral equation formulation is more effective computationally than the delay differential equation form. Finally we remark that initial conditions affect the well-posedness of the model, and we propose a modification to the mathematical model that ensures realistic values for a large class of initial conditions and contact rate parameter.

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Long-time behaviour of solutions to a nonlinear system related to a fluid flow model

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We study a nonlinear system containing three different types of differential equations: an ordinary, a parabolic and an elliptic one. The system is a generalization of a one-dimensional model describing fluid flow in porous media. We show existence of weak solutions in the time interval $(0, \infty)$, further, some asymptotic properties are investigated such as boundedness and stabilization of solutions as $t \rightarrow \infty$. The theoretical background of our investigations is the theory of monotone operators.

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Ordinary Differential Equations on Hilbert

Spaces Motivated by Learning Models

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James Jamison

We study a system of ordinary differential equations in $\mathcal{B}(\mathcal{H})$, the space of all bounded linear operators on a separable Hilbert space \mathcal{H} . The system considered is a natural generalization of the Oja-Adams learning models.

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Long time asymptotics for fast diffusion

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Herbert Koch and Robert McCann

Based on previous spectral results of two of the authors, we develop the convergence rate of fast diffusion towards the Barenblatt self-similar solution by means of semigroup and dynamical systems methods.

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Re-entrant corner flow of Phan-Thien-Tanner fluids

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We consider the local asymptotic behaviour for planar flow of Phan-Thien-Tanner (PTT) fluids around re-entrant corners, i.e. corners of angle π/α , where $1/2 \leq \alpha < 1$. We assume the situation of complete flow around the corner with the absence of lip vortices and consider the PTT model in the limit of vanishing solvent viscosity and for model parameter values $\kappa = O(1)$.

The asymptotic structure is shown to comprise an outer core flow region in which the upper convected stress derivative dominates together with narrow boundary layer regions at upstream and downstream walls at which viscometric behaviour is recov-

ered. A class of similarity solutions is derived for the outer core flow problem which give stress singularities of $O(r^{-2(1-\alpha)})$ (with r the radial distance from the corner) and a stream function behaviour of $O(r^{\alpha(1+\alpha)})$. These solutions are matched to wall boundary layers, the leading order equations of which being those associated with the high Weissenberg limit. Matching to the downstream wall boundary layer imposes a restriction on the validity of these solutions limiting them to corner angles for which $2/3 < \alpha < 1$.

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Stability and Approximation for Linear Systems

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We consider the linear Cauchy problem $x'(t) = Ax(t)$ on a Hilbert space X , and discuss the question of whether finite dimensional, semidiscrete approximations preserve the exponential stability of the solution semigroup. The question is investigated for applications involving linear delay-differential equations and linear partial differential equation models of certain damped elastic structures.

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Structured populations: Stability, immigration and the net growth rate

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Thomas Hagen

In this talk we propose structured population model(s) with an external inflow of individuals. We show that linear stability is governed by a generalized net reproduction function. We also demonstrate (through a concrete example, as well) how immigration might be beneficial to the population. In particular, we show that from a (nonlinearly) unstable positive equilibrium a linearly stable and unstable pair of equilibria bifurcates.

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Approximating the basin of attraction of time-periodic ODE's by meshless collocation of a Cauchy problem

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Holger Wendland

The basin of attraction of an equilibrium or periodic orbit can be determined by sublevel sets of a Lyapunov function. The main property of a Lyapunov function is its negative orbital derivative. We construct a Lyapunov function by approximately solving a Cauchy problem with a linear PDE for its orbital derivative using meshless collocation. This method has already been applied to autonomous dynamical systems.

In this talk, however, we consider the dynamical system given by a time-periodic ODE $\dot{x} = f(t, x)$, $x \in \mathbb{R}^n$, and assume that Ω is an asymptotically stable periodic orbit with basin of attraction $A(\Omega)$. We consider the (time-periodic) Lyapunov function satisfying

$$\begin{aligned} LV(t, x) &= -1 \text{ for } (t, x) \in A(\Omega) \setminus \Omega \\ V(t, x) &= 1 \text{ for } (t, x) \in \Gamma, \end{aligned}$$

where Γ is a non-characteristic set and the orbital derivative is given by $LV(t, x) = \langle \nabla_x V(t, x), f(t, x) \rangle + \partial_t V(t, x)$. Hence, V is the solution of a Cauchy problem with a linear first-order PDE. We approximate V by v using meshless collocation and derive error bounds on $|LV(t, x) - Lv(t, x)|$.

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Analytical advances in flows of free liquid fibers and films

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In this lecture we present recent advances in the mathematical theory of free liquid fibers and films of highly viscous liquids. The governing equations arise in the slender body approximation of the Stokes equations with free boundary as 1D or 2D nonlinear coupled hyperbolic-elliptic systems of pdes. Topics of interest include existence and uniqueness results, spectral determinacy and regularity of the linearized film equations, and failure of fiber breakup in the absence of surface tension.

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Some non-linear Differential Equations in spaces of Compact Operators

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Fernanda Botelho

Abstract: We study existence, uniqueness, and stability of a class of nonlinear differential equations in the space \mathcal{KH} of compact operators on a complex Hilbert space. The differential equations are motivated by models which arise in neural networks and have been introduced by Xu, and Chen, Amari, and Lin.

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Spatial Patterns Produced by Neural Field Equations

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The temporary storage and processing of information in the brain is a widely studied area of neuroscience. Research suggests that spatially patterned neural activity has a role in the short term encoding of information. We consider the integro-differential equation

$$\frac{\partial u(x, t)}{\partial t} = -u(x, t) + \int_{-\infty}^{\infty} \omega(x - y) f(u(y, t)) dy + h,$$

proposed in literature as model of neural activity. We investigate the existence of stationary “2-bump” solutions – functions that are positive on exactly two finite intervals, resembling localized neural activity. We assume that the coupling function ω is a generalized “Mexican hat” function.

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Finite Element Analysis of a Dynamic Contact Problem

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Janos Turi

In this presentation we describe a computational framework to approximate solutions of hyperbolic two-point boundary value problems with contact boundary conditions. As an application the influence of a crack on the dynamics of a cantilever beam

is investigated.

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On the extensional motion of a falling liquid sheet

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Thomas P. Witelski

We examine the extensional motion of an incompressible viscous planar sheet falling under the influence of gravity and derive a time-dependent exact solution from the full Navier-Stokes equations. The solution also satisfies the slender body equations for a long, thin liquid sheet. The linear stability of the time-dependent exact solution to one- and two-dimensional symmetric perturbations is examined in the inviscid and viscous limits within the framework of the slender body equations. Both transient growth and long-time asymptotic stability are considered. For one-dimensional perturbations in the axial direction, viscous and inviscid sheets are asymptotically marginally stable, though we find that transient growth can have an important effect and depends on the values of the Reynolds and Weber numbers. For one-dimensional perturbations in the transverse direction, inviscid sheets are asymptotically unstable to perturbations of all wavelengths. For two-dimensional perturbations, inviscid sheets are unstable to perturbations of all wavelengths with the transient dynamics controlled by axial perturbations and the long-time dynamics controlled by transverse perturbations. The asymptotic stability of viscous sheets to one-dimensional transverse perturbations and to two-dimensional perturbations depends on the capillary number (Ca); in both cases, the sheet is unstable to longwave transverse perturbations for any finite Ca.

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Stochastic Variational Inequalities for Elasto-Plastic Oscillators

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Alain Bensoussan

A stochastic variational inequality is proposed to model a white noise excited elasto-plastic oscillator.

The solution of this inequality is essentially a continuous diffusion process for which a governing diffusion equation is obtained to study the evolution in time of its probability distribution. The diffusion equation is degenerate, but using the fact that the degeneracy occurs on a bounded region we are able to show the existence of a unique solution satisfying the desired

properties. We prove the ergodic properties of the process and characterize the invariant measure. Our approach relies on extending Khasminskii's method, which in the present context leads to the study of degenerate Dirichlet problems with nonlocal boundary conditions.

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