Special Session 25: Long Time Behavior of Hamiltonian and Dissipative Systems

Atanas Stefanov, The University of Kansas, USA
Milena Stanislavova, The University of Kansas, USA

On Stability of steady-state solutions for the Navier-Stokes equation.

Clayton Bjorland
University of California, Santa Cruz, USA
cbjorland@math.ucsc.edu

In this talk we consider the stability of steady-state Navier-Stokes equation in the whole space $\mathbb{R}^3$ driven by a forcing function f. We will describe some conditions on the forcing function f which ensure perturbed systems eventually return to the original steady-state.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Existence and Sability of Travelling Waves for a Class of Nonlinear Dispersive Equations

Hongqiu Chen
University of Memphis, USA
hchen1@memphis.edu

Considered here is a general class of nonlinear, dispersive wave equations

$$u_t - Lu_x + f(u)_x = 0, \quad x \in \mathbb{R}, \quad t \geq 0, \qquad (1)$$

where f is a nonlinear function, typically, a polynomial with $f(0) = f'(0) = 0$ and L is the dispersion operator defined through its Fourier symbol α, say. Thus, L and α are related by

$$\widehat{Lv}(\xi, t) = \alpha(2\pi\xi)\widehat{v}(\xi, t)$$
$$\text{where} \quad \widehat{v}(\xi, t) = \int_{-\infty}^{\infty} v(x, t)e^{-2\pi i\xi x}\, dx$$

for all wavenumbers ξ. The symbol α is taken to be a real, even continuous function vanishing at the origin and becoming unbounded as $\xi \to \pm\infty$. Theory on existence and orbital stability of solitary wave solutions and cnoidal wave solutions are established.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Existence of global attractor for a periodic Kuramoto-Sivashinsky type

Aslihan Demirkaya
University of Kansas, USA
ademirkaya@math.ku.edu

We consider a periodic Kuramoto-Sivashinsky type equation in space dimension two. We use a Lyapunov function argument to show the boundedness of the solution for large times. We prove the existence of global attractor in $L^2([-L; L]^2)$ when the equation is subject to L^2 external force.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Long time behavior of damped Boussinesq systems

Olivier Goubet
LAMFA UMR 6140 CNRS, Universite de Picardie, France
olivier.goubet@u-picardie.fr
Min Chen (Purdue)

In this talk, we consider the two-dimensional dissipative Boussinesq systems which model surface waves in three space dimension. The long time asymptotics of the solutions for a large class of such systems are obtained rigorously for small initial data. Some numerics are performed. This work is performed with the support of CNRS (Research Program "Water-waves")

$\longrightarrow \infty \diamond \infty \longleftarrow$

The global attractor for the solutions of the viscous Primitive Equations.

Ning Ju
Oklahoma State University, USA
ningju@math.okstate.edu

The system of 3D viscous Primitive Equations models the large scale motions of ocean and atmosphere. We discuss the problem of the global attractor for the 3D viscous Primitive Equations, its existence and its regularities.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Regularity and uniform gradient bounds for solutions of the Primitive Equations of the

Ocean

Igor Kukavica
University of Southern California, USA
kukavica@usc.edu
Mohammed Ziane

We prove the existence of global strong solutions of the primitive equations of the ocean in the case of the Dirichlet boundary conditions on the side and the bottom boundaries and provide uniform gradient bounds.

⟶ ∞ ◇ ∞ ⟵

Orbital stability in a combustion problem

Yuri Latushkin
University of Missouri-Columbia, USA
yuri@math.missouri.edu

We study a nonlinear partial differential equation that appears in a combustion problem, and prove the orbital stability of the corresponding front. The analysis involves the Evans function, spectral mapping theorems for linear strongly continuous semigroups, and a'priory estimates. This is a joint project with Anna Ghazaryan and Steve Schecter.

⟶ ∞ ◇ ∞ ⟵

Justification of tight-binding approximation for space-periodic problems

Dmitry Pelinovsky
McMaster University (Canada), Canada
dmpeli@math.mcmaster.ca

We justify the validity of the discrete nonlinear Schrödinger equation for the tight-binding approximation in the context of the Gross-Pitaevskii equation with a periodic potential. We rely on properties of the Floquet band-gap spectrum and the Fourier-Bloch decomposition for a linear Schrodinger operator with a periodic potential. Our analysis is valid for a class of piecewise-constant periodic potentials with disjoint spectral bands, which reduce, in a singular limit, to a periodic sequence of infinite walls of a finite width. The discrete nonlinear Schrodinger equation is applied to classify localized solutions of the Gross–Pitaevskii equation with a periodic potential. Time evolution of localized solutions is studied on large but finite time intervals using analysis involving energy estimates and Gronwall's inequality.

⟶ ∞ ◇ ∞ ⟵

On the energy of singular inviscid flows

Roman Shvydkoy
University of Illinois at Chicago, USA
shvydkoy@math.uic.edu

The energy conservation law is an inherent property of classical Hamiltonian systems. In the case of the Euler equation solutions are required to possess a certain minimal degree of regularity predicted by Ondager to obey this law. We will discuss validity of the energy conservation for Onsager-critical singular weak solutions to the Euler equations. We show that if the singular set of the solution is located on a regular low-dimensional family of submanifolds, or forms a smooth family of orientable hypersurfaces (slits), then the energy is conserved. As a consequence all classical 3D and 2D (with zero total circulation) vortex sheets are energy conservative. From the viewpoint of turbulence one may argue that Kolmogorov-type solutions to the Navier-Stokes equation cannot form organized families of singular sets in the limit of vanishing viscosity, as those solutions are likely to possess anomalous dissipation due to non-vanishing energy flux.

⟶ ∞ ◇ ∞ ⟵

Refined Gevrey estimates for the Kuramoto-Sivashinsky equation

Milena Stanislavova
University of Kansas, USA
stanis@math.ku.edu
A. Stefanov

We consider the Kuramoto-Sivashinsky (KS) equation in finite domains. Our main result provides logarithmic refinement of the Gevrey estimates for the solution of the one dimensional differentiated KS equation. We also show local well-posedness for the two dimensional KS equation and provide an explicit criteria for (eventual) blow-up in terms of the L^2 norm.

⟶ ∞ ◇ ∞ ⟵

Random Attractors for the Stochastic FitzHugh-Nagumo System on Unbounded Domains

Bixiang Wang
New Mexico Tech, USA
bwang@nmt.edu

We prove the existence of a compact random attractor for the stochastic FitzHugh-Nagumo system defined on the entire space $\mathbb{R}^n$. This random attractor attracts every pulled-back tempered random set under the forward flow. The asymptotic compactness of the random dynamical system is established by uniform estimates on the tails of solutions.

$\longrightarrow \infty \diamond \infty \longleftarrow$

The 2D dissipative surface quasi-geostrophic equation

Jiahong Wu
Oklahoma State University, USA
jiahong@math.okstate.edu

We will report recent results on the issue of global existence and uniqueness of solutions to the dissipative surface quasi-geostrophic (QG) equation and modified surface QG equation. The recent progress by Kiselev, Nazarov and Volberg and by Caffarelli and Vasseur on the critical case will be briefly described and the major results of Constantin and Wu's two papers on the supercritical case will be detailed. In addition, the global regularity result of Constantin, Iyer and Wu on a modified critical dissipative QG equation will be presented.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Global Dynamics of 3D Gray-Scott Equations

Yuncheng You
University of South Florida, USA
you@math.usf.edu

Based on the recent result of the existence of a global attractor for Gray-Scott equations shown by this author, it is proved further that there exists an inertial set with finite fractal dimension for the 3D Gray-Scott equations on a bounded domain. Alternative proofs are given by showing the squeezing property and by a compact and contracting decomposition of the Fréchet derivative of the solution semigroup.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Invariant manifold of dynamic spike solutions to a singular parabolic equation

Chongchun Zeng
Georgia Institute of Technology, USA
zengch@math.gatech.edu
Peter Bates and Kening Lu

Consider a nonlinear parabolic equation $u_t = \varepsilon^2\Delta u - u + f(u)$ on a smooth bounded domain $\Omega \subset \mathbb{R}^n$ with the zero Neumann boundary condition. In the past years, there had been extensive studies on steady spike solutions. Here a spike solution u is one which is almost equal to zero everywhere except on a ball of radius $O(\epsilon)$ where $u = O(1)$. In this talk, we show that there exist dynamic spike solutions which maintain the spike profile for all $t \in \mathbb{R}$ with the spike moving on $p\Omega$. Moreover, these dynamic spike states form an invariant manifold in the function space which is diffeomorphic to $\partial\Omega$.

$\longrightarrow \infty \diamond \infty \longleftarrow$