Special Session 26: Qualitative Analysis of Parabolic Equations

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Asymptotic behaviour of a multidimensional moving boundary problem

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Of concern is a moving boundary problem modelling the growth of multicellular spheroids or in vitro tumors. This model consists of two elliptic equations describing the concentration of a nutrient and the distribution of the internal pressure in the tumor's body, respectively. The driving mechanism of the evolution is governed by Darcy's law. Finally surface tension effects on the moving interface counteract the internal pressure. Based on a centre manifold analysis, we prove that if the initial domain is sufficiently close to a Euclidean ball in a suitable Hoelder norm, then the solution exists globally and the corresponding domains converge exponentially fast to some (possibly translated) ball, provided the surface tension coefficient is larger than a positive threshold value γ^*. If the surface tension coefficient is less than γ^* then the radially symmetric equilibrium centred at the origin is unstable.

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Analysis of a reaction-diffusion system: Dynamics and steady states

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In this talk, we will discuss our recent results on Gierer-Meinhardt system. One of the main difficulties of the system is the singular structure of its reaction term, we will show how new techniques help to overcome such difficulty.

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A variational model for microstructure generation

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We analyze the minimization problem

$$\min_{u\in W^{2,2}(0,1)} F^\epsilon(u)+\frac{\mu}{2\epsilon^2}\int_{(0,1)}(u-\bar{u})^2dx, \qquad (1)$$

where $\bar{u}$ is a given function, $0<\mu$, $0<\epsilon<<1$ are parameters and $F^\epsilon(u) := \int_{(0,1)}(\frac{\epsilon^2}{2}u_{xx}^2+\phi(u_x))dx$. We are interested in the case of a non convex energy density $\phi : (-\infty,+\infty)\to[0,+\infty)$. Numerical simulations indicate that, in time scales of $O(\epsilon^2)$, the gradient system defined by F^ϵ

$$\begin{aligned} u_t &= -\epsilon^2 u_{xxxx}+(\dot{\phi}(u_x))_x, \qquad (2)\\ +BC &\quad , \quad u(.,0)=\bar{u}, \end{aligned}$$

generates a fine microstructure in certain subregions of $(0,1)$ that are determined by the initial datum $\bar{u}$. The parameter μ in (1) can be interpreted as the inverse of a slow time τ defined by $t=\epsilon^2\tau$. Therefore one can expect that, for small $\epsilon>0$, minimizers $u^{\epsilon,\mu,\bar{u}}$ of (1) exhibit a structure similar to the one shown, for $\tau=\frac{1}{\mu}$, by the solution $u(.,\epsilon^2\tau,\bar{u})$ of (2) in the numerical experiments. We present several results that confirm the above expectation.

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Stability of the Planar Front in a Diffusive Free Boundary Problem

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In this talk we shall consider a Free Boundary Problem describing anomalous diffusion. Particular attention will be paid to the stability analysis of planar fronts, one dimensional solutions, with respect to two dimensional perturbations. The analysis is performed in a periodic context via a boundary integral formulation and the use of maximal regularity.

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Regularity of solutions to strongly coupled parabolic systems

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We consider strongly coupled parabolic systems consist of 2 equations given on n dimensional domains with n larger than two. Under suitable conditions on the diffusion matrix it can be proved that bounded weak solutions are Holder continuous. We also report recent results of this type for systems consist of more than two equations.

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Oblique derivative problems for parabolic equations

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We study problems of the type $u_t = F(x,t,u,Du,D^2u)$ in some space-time domain Ω with boundary condition $G(x,t,u,Du) = 0$ on the lateral boundary of Ω and prescribed initial data. Our interest is primarily with weak regularity hypotheses on the domain Ω and the function G. As a particular case, we mention the normal derivative problem on a cylindrical domain with piecewise smooth cross-section.

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On the shape of the stable pattern for activator-inhibitor systems in a disk

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We study the shape of the stable steady state to shadow systems of the activator-inhibitor type. Specifically, we show that if stable, then the shape should be like a boundary one-spike layer. We will see that this problem is deeply related to the "hot spots" conjecture of J. Rauch.

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Dynamics of a density discontinuity in compressible viscous flows. Contact problem.

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We will discuss the evolution of the interface of a density discontinuity in the model of Navier-Stokes equations for compressible flows. All time existence of weak, near equilibrium solutions will be established, when the interface is in contact with the boundary of the flow domain.

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Asymptotic symmetry of positive solutions of parabolic equations and systems

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We consider parabolic equations on (possibly nonsmooth) domains with reflectional symmetries. Under suitable conditions on the nonlinearity it can be proved that bounded positive solutions are asymptotically symmetric as time approaches infinity. We shall review recent results of this form for scalar equations and systems on bounded domains.

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Diffusive synchronization of spatially extended oscillations

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We will discuss properties of temporal oscillations in reaction-diffusion systems posed on $\mathbf{R}^n$. The main result shows that localized perturbations to initial conditions decay diffusively. We also discuss temporal asymptotics of bounded perturbations. We will conclude with some open problems concerned with the description of perturbations with bounded phase gradient.

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On the the Stefan problem with surface tension

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In this talk, I consider the two-phase Stefan problem with the modified Gibbs-Thomson law

$$u = \sigma H + \delta V \quad \text{on} \ \ \Gamma(t), \quad \sigma > 0, \ \delta \geq 0,$$

and the kinetic condition

$$[d\partial_\nu u] = (\ell - [\kappa]u)V \quad \text{on} \quad \Gamma(t).$$

Here $\Gamma(t)$ denotes the unknown moving hypersurface that separates the liquid from the solid phase, u is the temperature, H the mean curvature of $\Gamma(t)$, σ the surface tension coefficient, δ the coefficient of kinetic undercooling, V the normal velocity of $\Gamma(t)$, ℓ the latent heat, $[\kappa]$ the jump of the heat capacities across $\Gamma(t)$, and $[d\partial_\nu u]$ the jump of the heat fluxes across $\Gamma(t)$.

It is shown that this system conserves energy, and admits a Ljapunov function. Under appropriate boundary conditions we also show that spheres (together with constant temperature distributions) are the only equilbrium states for this system, and we will characterize the stability of these equilibria in dependence of physical and geometric quantities.

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An age and spatially structured population model for Proteus mirabilis swarm-colony development

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Proteus mirabilis are bacteria that make strikingly regular spatial-temporal patterns on agar surfaces. In this talk we consider a mathematical model that has been shown to display these structures when solved numerically. The model consists of an ordinary differential equation coupled with a partial differential equation involving a first-order hyperbolic aging term together with nonlinear degenerate diffusion. We prove that the system admits global weak solutions.

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Solutions with moving singularities for a semilinear parabolic equation

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We consider the Cauchy problem for a semilinear parabolic equation with a power nonlinearity. It is known that the equation has a singular steady state in some parameter range. Our concern is the existence of a solution with a moving singularity that is obtained by perturbing the singular steady state. By the formal expansion, it turns out that the correction term must satisfy a parabolic equation with inverse-square potential. From the well-posedness of this equation, we see that there appears a critical exponent. Paying attention to this exponent, given a motion of the singular point and suitable initial data, we establish the time-local existence, uniqueness and comparison results for a solution with a moving singularity for a certain class of initial data.

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Strong non-collapsing and uniform Sobolev inequality for Ricci flow with surgeries

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We prove a uniform Sobolev inequality for Ricci flow, which is independent of the number of surgeries. As an application, under less assumptions, a non-collapsing result stronger than Perelman's κ non-collapsing with surgery is derived. The proof is much shorter and seems more accessible. The result also improves some earlier ones where the Sobolev inequality depended on the number of surgeries.

Another application is a proof of Hamilton's little loop conjecture win case of surgeries. The conjecture was proved by Perelman in the case without surgery.

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