Special Session 27: Sign-changing Solutions for Nonlinear Elliptic Problems

Dumitru Motreanu, , France

On a class of nonlinear boundary value problems

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We establish the existence of multiple nontrivial solutions (including a nodal one) for a class of superlinear Neumann problems associated with the p-Laplacian. The approach relies on a combination of variational arguments, truncation techniques, and the method of upper-lower solutions. This is part of a recent joint work with N. S. Papageorgiou and V. Staicu.

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Some abstract existence theorems and applications to p–Laplacian type problems

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Since the Palais' pioneer paper in 1963, condition (C) in both Palais–Smale and Cerami's Palais–Smale version, has been widely used in order to prove minimax existence theorems for C^1 functionals in Banach spaces.

Recently, a generalization of Cerami's Palais–Smale condition has been introduced so that a quantitative deformation lemma holds and some existence theorems can still be obtained.

Such abstract results apply to p–Laplacian type elliptic problems.

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Nodal and Multiple Constant Sign Solutions for p-Laplacian Equations at Resonance

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We study a nonlinear Dirichlet elliptic differential equation driven by the p-Laplacian and with a nonsmooth potential. Using a variational approach combined with suitable truncation techniques and the method of upper-lower solutions, we prove the existence of five nontrivial smooth solutions, two positive, two negative and the fifth nodal. Our hypotheses on the nonsmooth potential allow resonance at infinity with respect to the principal eigenvalue.

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A general positivity condition for variatiobal inequalities

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K. Addi

Using recession tools like recession cones and recession functions, we first introduce a new class of problems that we call "semi-complementarity problem". Then we will show that the study of a large class of variational inequalities of the second kind can be reduced to the one of a generalized eigenvalue problem governed by some semi-complementarity problem. In this talk we will present the methodology and the main theoretical results. Applications in electronics will be discussed by K. Addi in session 9.

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Lyapunov-Schmidt reduction, critical groups and mulltiple solutions of elliptic resonant problems

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We discuss an approach by combining the homotopy invariance, Lyapunov-Schmidt reduction method and Alexander Duality Theorem to compute the critical groups of functionals corresponding to elliptic boundary value problems. Then, by using Morse theory, truncated technique and the three critical point theorem, we obtain multiple solutions for elliptic resonant problems. Here, the corresponding functional does not satisfy the (PS) condition. This difficulty is overcame by taking advantage of the Lyapunov-Schmidt reduction and a careful analysis of the reduced functional.

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Variational asymptotic method for homogenization of boundary hemivariational inequalities

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We deal with a mathematical model describing stationary elastic contact problems with the Hooke constitutive law and subdifferential boundary conditions. The problem is formulated in a weak form as a boundary hemivariational inequality. The existence and uniqueness of solutions to hemivariational inequality is established. Using the notion of H-convergence of elasticity tensors we investigate the limit behavior of the sequence of solutions to hemivariational inequalities. From the mechanical point of view, the asymptotic analysis determines the large scale properties of the material without determining its fine scale structure. The limit homogenized tensor defines an effective properties of the medium.

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Multiple and sign-changing solutions for nonlinear elliptic problems

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One obtains multiple solutions for a class of parametric (p-1)-superlinear Dirichlet problems involving the p-Laplacian. Specifically, under suitable hypotheses, we prove that if the parameter is sufficiently small there exist at least six nontrivial solutions for which we are able to provide sign information. Namely, there are two positive solutions, two negative solutions and two nontrivial sign-changing solutions.

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A class of dynamic hemivariational inequalities modeling fully nonlinear viscoelastic contact problems

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We consider a class of abstract second order evolution inclusions with nonlinear operators depending on the unknown function and its time derivative and involving a nonmonotone and multivalued subdifferential mapping. We prove the existence and uniqueness result. The proof is based on arguments of evolutionary hemivariational inequalities with monotone operators and the Banach fixed point theorem. The result is applicable to a mathematical model of frictional contact in the theory of nonlinear viscoelasticity.

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Multiplicity of solutions for parametric p-Laplacian equations

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We consider a nonlinear elliptic problem driven by the p-Laplacian and depending on a parameter $l > 0$. The right hand side nonlinearity is concave, (i.e., p-sublinear) near the origin. For such problems we prove two multiplicity results. One when the right hand side nonlinearity is p-linear near infinity and the other when it is p-superlinear. Both results show there exists $\hat{l}_0 > 0$ such that the problem has five nontrivial solutions (two positive, two negative and one nodal), if the parameter $l \in (0, \hat{l}_0)$. We also consider the case $l = \hat{l}_0$.

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Sign-changing solutions for an asymptotically linear Schrödinger equation

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Let us consider the following Schrödinger equation

$$\begin{cases} -\Delta u + V(x)u = f(x,u) & x \in \mathbf{R}^N \\ u(x) \to 0 \quad \text{as } |x| \to \infty \end{cases}$$

where the potential $V(x)$ is such that the linear operator $-\Delta + V(x)$ has a discrete spectrum in $L^2(\mathbf{R}^N)$.

Taking a suitable asymptotically linear nonlinearity $f(x,u)$, we prove that the problem has either a sign-changing solution or one positive and one negative solution.

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On the uniqueness of nodal radial solutions of sublinear elliptic equations in a ball

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The following Dirichlet problem

$$(1)\qquad \begin{cases} \Delta u + K(|x|)f(u) = 0 & \text{in } B, \\ u = 0 & \text{on } \partial B, \end{cases}$$

is considered, where $B = \{x \in \mathbf{R}^N : |x| < 1\}$, $N \geq 3$, $K \in C^2[0,1]$ and $K(r) > 0$ for $0 \leq r \leq 1$, $f \in C^1(\mathbf{R})$, $sf(s) > 0$ for $s \neq 0$. Assume moreover that f satisfies the following sublinear condition: $s^{-1}f(s) > f'(s)$ for $s \neq 0$. A sufficient condition is derived for the uniqueness of radial solutions of (1) possessing exactly $k-1$ nodes, where $k \in \mathbf{N}$. It is also shown that there exists $K \in C^\infty[0,1]$ such that (1) has three radial solutions possessing exactly $k-1$ nodes, where $k \geq 2$. These results are obtained from an elementary shooting method. However it seems that they are technical and nontrivial.

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Sign-changing and multiple solutions of Kirchhoff type problems

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K. Perera

We obtain nontrivial solutions of a class of nonlocal quasilinear elliptic boundary value problems using the Yang index and critical groups. We also obtain sign changing solutions of a class of nonlocal quasilinear elliptic boundary value problems using variational methods and invariant sets of descent flow (even for the case without P.S. condition).

$$\begin{cases} -\left(a + b\int_\Omega |\nabla u|^2\right)\Delta u = f(x,u) & \text{in } \Omega \\ u = 0 & \text{on } \partial\Omega \end{cases} \qquad (1)$$

where Ω is a smooth bounded domain in R^n, $n = 1, 2$, or 3, $a, b > 0$, and f is a Carathéodory function on $\Omega \times R$.

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