Special Session 28: Fluids and Turbulence

Animikh Biswas, USA

3D Navier-Stokes equations in time varying Gevrey spaces

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Ever since the work of Foias and Temam in case of the Navier-Stokes equations (NSE), the use of Gevrey norms has become standard fare in estimating the radius of analyticity in various nonlinear evolution equations. Recent investigations revealed that these are also closely connected to asymptotic behavior of solutions and the study of the global attractor. In most of the studies to date, by considering an adequate Gevrey norm, one shows that the space analyticity radius grows at least as a power of time upto a certain time point. This time point is usually expressed in terms of a Sobolev norm of the initial data. On the other hand, the solution to the 3D NSE is known to be analytic as long as its Sobolev $\mathbb{H}^1$ norm is finite. We consider an auxilliary equation that couples the the space analyticity radius to the 3D NSE. This allows us to study its time evolution more precisely and thus obtain improved estimates. As a corollary, we show that a close relation exists between the analyticity radius and the regularity of the solution to the 3D NSE.

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Effects of scales on rotational Euler equations

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We study the stabilizing effect of rotational forcing in the nonlinear setting of two-dimensional shallow-water and Euler equations. Two distinct scaling regimes are studied. When rotational force dominates the pressure, it prolongs the life-span of smooth solutions for $t \lesssim 1 + \ln(\delta^{-1})$ where δ is the ratio of the (inverse of) squared Froude number measuring the amplitude of pressure, relative to the (inverse of) Rossby number, measuring the dominant rotational force. The strong rotation also imposes certain periodicity to the flow in the sense that there exists a "nearby" periodic-in-time approximation of the exact solution. In the opposite regime of large δ, the flow is dispersive so that the divergence field substantially decays in finite time and therefore periodicity is not retained.

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On the regularity of weak solutions to the 3D Navier-Stokes equations in Besov spaces

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We will discuss some new results on the regularity of weak solutions to the 3D Navier-Stokes equations in Besov spaces.

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Behavior of average energy and enstrophy of turbulent flows for big Grashof numbers

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We will use Navier-Stokes equations to derive asymptotic behavior (with respect to the Grashof number) of the average energy and average enstrophy of the 3D flows in both general and turbulent cases.We will discuss the implications of these bounds to the strength of energy cascades and to the behavior of the nonlinear term. This is joint work with C. Foias and M. Jolly.

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A search for a Shangri-la norm for the 3D Navier-Stokes equations

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The existence times of solutions to the 3D NSE in the Sobolev space H^s, expressed in terms of the $\mathbb{H}^s$ norm of the initial data, improve with increasing s, i.e., the existence times improve as the spaces decrease. On the other hand, there is a family of Gevrey spaces, which are included in the intersection of all

the Sobolev spaces $\mathbb{H}^s$, for which the existence times improve as the spaces increase. A Shangri-la norm is a norm in between the norms of these two families for which the existence time is longest, perhaps infinity. We will show that there are spaces that lie in between the Sobolev and the Gevrey families for which the time of existence is larger than in either family.

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The role of geometry/topology of the magnetic field in fusion plasmas MHD turbulence

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One of the most important issues in understanding fusion plasmas MHD is understanding MHD turbulence-driven anomalous transport. In particular, the role of geometry/topology of the magnetic (confinement) field in depletion of the nonlinear effects and self-regulation of turbulence. Some recent results pertaining toroidal (tokamak-type) geometries/coherent structures (e.g., streamers) will be presented.

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Truncation Errors on Spectral Methods of the Shallow Water Equations

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In this talk, we study the truncation errors introduced by applying double Fourier and Legendre-Fourier transformations to the Shallow water equations on the sphere. The comparison of the truncation errors is based on the terms in the Sallow water equations. Also we investigate the impact of the truncation error on the variable resolution for the Shallow water equations on the sphere. The numerical results show the comparison of the truncation errors after employed a conformal mapping for both transformations.

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On the normal form of the Navier-Stokes equations in suitable Banach spaces

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Ciprian Foias, Eric Olson and Mohammed Ziane

We consider solutions to the incompressible Navier-Stokes equations on the periodic domain $\Omega = [0, 2\pi]^3$ with potential body forces. Let $\mathcal{S} \subseteq H^1(\Omega)^3$ denote the set of all initial data that lead to regular solutions. Our main result is to construct a suitable Banach space $\S^s_A tar$ such that the normalization map $W : \mathcal{S} \to \S^s_A tar$ is continuous, and such that the normal form of the Navier-Stokes equations is a well-posed system in all of $\S^s_A tar$. We also show that $\S^s_A tar$ may be seen as a subset of a larger Banach space $V^\star$ and that the extended Navier-Stokes equations, which are known to have global solutions, are well-posed in $V^\star$.

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2-D turbulence estimates for general forces

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N. Balci and C. Foias

Three rigorous estimates to support the Batchelor-Kraichnan-Leith theory of 2-D turbulence are generalized to the case of time-dependent forcing at all length scales. The first bounds the dissipation wavenumber from above and below in terms of a generalized Grashof number. The second relates a averaged Dirichlet quotient to the dissipation wavenumber, and the third provides one side of the dissipation law. These estimates impose certain restrictions on the shape of the force.

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Dissipativity in Models of Quasi-Geostrophic Turbulence

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Numerical simulations of forced-dissipative "2-1/2 ?[4]? dimensional" models of quasi-geostrophic (QG) turbulence suggest the existence of finite dimensional attractors for the underlying coupled pairs of 2-d PDEs. Certain features of the models and their numerical solutions suggest an analogy with a simpler pattern-forming model, the 1-d Kuramoto-Sivashinsky equation. Ideas used in the study of the KS equation are used to demonstrate dissipativity of the QG systems, and to give bounds on attractor dimensions. The talk will sketch the character of numerical solutions that make them geophysically interesting and outline ideas in the proof of dissipativity.

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A regularity criterion for the dissipative quasi-geostrophic equations

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Hongjie Dong

In this talk we will present the joint work with Hongjie Dong concerning a regularity criterion for weak solutions of the dissipative quasi-geostrophic equations. We remark that the result extends the regularity criterion of Constantin and Wu to scaling invariant spaces.

After we give a short overview of the known results concerning the 2D quasi-geostrophic equations, we will present a sketch of the proof of the regularity criterion.

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Couplings of PDE/stochastic systems

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Markos Katsoulakis, Andrew Majda and Dion Vlachos

We undertake a systematic study of examining hybrid systems consisting of partial differential equations (PDEs) coupled to stochastic lattice models. The coupling of the stochastic model arises as a boundary contribution to the PDE while at the same time the PDE acts on the stochastic model as an external, although local, forse via its interaction potential.

Specifically the stochastic component includes both spin-exchange (with/out look- ahead) and spin-flip Arrhenius dynamics and we systematically study the system behavior by ranging the parameters responsible for each mechanism. Therefore we can isolate and study effects originating from just one stochastic mechanism or a combination of both.

In this manner we explore the hybrid system via a multi-time scale Markov jump process. Through this study, the key question of significant preparation created in the model through the look-ahead rules can also be explored. Kinetic Monte Carlo simulations are undertaken to facilitate the results.

Applications of such systems range from chemical catalysis to climate prediction and forecasting.

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Local Well-Posedness of A Dispersive Navier-Stokes System

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C. David Levermore

Local smoothing effect and well-posedness of a degenerate dispersive Navier-Stokes system are studied. This system comes from the formal asymptotic analysis of kinetic equations. Under the assumptions that the initial data satisfy asymptotic flatness and non-trapping conditions, we show that there exists a unique classical solution to this system in a finite time interval.

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A free boundary problem for a parabolic equation satisfying an integral condition

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Changbing Hu and Yongzhi Xu

We consider the well-posedness of a parabolic initial/boundary value problem which satisfies an integral condition on the free boundary. This problem is motivated by a model of breast cancer which mimics the growth of a ductal carcinoma in situ.

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Inertial manifolds for a nonlinear Fokker-Planck equation

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A nonlinear Fokker-Planck equation (Smoluchowski equation) arises in modeling of nematic polymers.

Coupling with fluid equations - or even a simple passive advection by a shear flow - leads to a complicated and peculiar dynamical behavior. The equation in its original form does not satisfy the spectral gap condition; the equation, however, can be transformed so that the classical theorems apply.

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Numerical simulation of three-dimensional incompressible fluid in spectral accuracy

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A collocation spectral numerical scheme for the three-dimensional (3-D) incompressible Navier-Stokes equations is proposed and studied in detail. The velocity vector field is dynamically updated by Runge-Kutta method, with the pressure gradient being treated as a force term instead of a Lagrange multiplier. Subsequently, the pressure is determined by the velocity vector through a pressure Poisson equation. To avoid a numerical boundary layer, a consistent and spectrally accurate boundary condition for the pressure is derived, by utilizing a Hermite interpolation for the normal velocity around the boundary. The overall scheme is proven to be highly efficient, stable and capable of producing accurate results with a reasonable computational cost. A perfect spectral accuracy is observed in the accuracy check. In addition, numerical simulations of three physical examples are also presented, including Poiseuille flow, 3-D lid driven cavity flow and 3-D differentially heated cavity.

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On the global regularity of sub-critical Euler-Poisson equations

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We prove that the one-dimensional Euler-Poisson system driven by the Poisson forcing together with the usual γ-law pressure, $\gamma \geq 1$, admits global solutions for a large class of initial data. Thus, the Poisson forcing regularizes the generic finite-time breakdown in the 2×2 p-system. Global regularity is shown to depend on whether or not the initial configuration of the Riemann invariants and density crosses an intrinsic critical threshold.

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