

## Special Session 29: Nonlocal Equations and Diffusion Problems

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### a non monotone nonlocal PDE for dune morphodynamics.

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We study a nonlocal equation introduced by A.C. Fowler describing dune morphodynamics. We prove local existence in a suitable function space and prove also existence of traveling waves.

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### Nonlocal Problems for Parabolic Inclusions

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Let  $\Omega$  be an open bounded domain in  $\mathbb{R}^N$ , and  $T > 0$ . We are concerned with the existence of solutions of the following parabolic inclusion

$u_t + Lu \in F(x, t, u)$ ,  $(x, t) \in \Omega \times (0, T)$ ,  $u(x, t) = 0$  on  $\partial\Omega \times [0, T]$

subjected to the nonlocal condition  $u(x, 0) = \int_0^T g(x, t, u(x, t))dt$ ,  $x \in \Omega$ .

We provide sufficient conditions on  $L$ ,  $F$  and  $g$  that guarantee the existence of at least one solution.

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### Optimal decay rate estimates for viscoelastic dissipative

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The linear viscoelastic equation is considered:

$$\begin{cases} y_{tt} - \Delta y + h * \Delta y = 0 & \text{in } \Omega \times (0, \infty) \\ y = 0 & \text{on } \Gamma_0 \times (0, \infty) \\ \partial_\nu y - h * \partial_\nu y + g(y_t) = 0 & \text{on } \Gamma_1 \times (0, \infty) \end{cases}$$

where  $\Omega$  is a bounded domain of  $\mathbb{R}^n$ ,  $n \geq 1$ , with a smooth boundary  $\Gamma = \Gamma_0 \cup \Gamma_1$ . Here,  $\Gamma_0$  and  $\Gamma_1$  are

closed, non-empty and disjoint and  $\nu$  represents the unit outward normal to  $\Gamma$ .

We prove uniform decay rates of the energy by assuming a nonlinear feedback acting on the boundary, *without imposing any growth assumption on the damping term near the origin and strongly weakening the usual assumptions on the domains, namely, star-shaped ones, when  $-\frac{h'}{h}$  is bounded from below by a positive constant and improving the usual assumptions on the kernels.* Our estimate depends both on the behavior of the damping term near zero and on the behavior of the relaxation function at infinity.

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### Runge-Kutta convolution quadrature methods for well-posed equations with memory

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**M. P. Calvo and C. Palencia**

In this work we consider well-posed linear equations with memory  $u(t) = A * u(t) + f(t)$ ,  $t \geq 0$ , and its time discretization by means of Runge-Kutta convolution quadrature methods. In this setting  $A(t)$  stands for family of linear operators with suitable assumption concerning with the decreasing behaviour at infinity. We notice that the sectorial case already studied (see [Lubich-Ostermann, 93]) The main result of this work shows that the numerical solution yielded by the mentioned methods can be written as a rate of the continuous solution integrated along a positive measure. From this results, the error analysis turns out to be straightforward by applying very well know results of [Brenner-Thomme, 79] related with rational approximations of semigroups.

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### Asymptotic stability of the wave equation on compact surfaces

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**Marcelo Cavalcanti, Ryuichi Fukuoka and Juan Soriano**

This talk is concerned with the study of the wave

equation on compact surfaces and locally distributed damping, described by

$$u_{tt} - \Delta_{\mathcal{M}}u + a(x)g(u_t) = 0 \quad \text{on } \mathcal{M} \times ]0, \infty[,$$

where  $\mathcal{M} \subset \mathbb{R}^3$  is a smooth oriented embedded compact surface without boundary. Denote the Riemannian metric induced on  $\mathcal{M}$  by  $\mathbb{R}^3$  by  $\mathbf{g}$ . First of all we prove that for  $\epsilon > 0$ , there exist an open subset  $V \subset \mathcal{M}$  and a smooth function  $f : \mathcal{M} \rightarrow \mathbb{R}$  such that  $meas(V) \geq meas(\mathcal{M}) - \epsilon$  and  $Hess f \approx \mathbf{g}$  on  $V$ .

Finally we prove that if  $a(x) \geq a_0 > 0$  on an open subset  $\mathcal{M}^* \subset \mathcal{M}$  that contains  $\mathcal{M} \setminus V$  and if  $g$  is a monotonic increasing function such that  $k|s| \leq |g(s)| \leq K|s|$  for all  $|s| \geq 1$ , then uniform and optimal decay rates of the energy hold.

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### Nonlocal Reaction-Diffusion Problems from Climate

**Georg Hetzer**

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Of concern are (degenerate) reaction-diffusion systems with memory terms on the 2-sphere. Such problems arise from diagnostic climate models with bio-feedbacks. We discuss global solvability and long-term behavior.

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### Elliptic Equations with Nonlocal Diffusion

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We will consider an elliptic boundary value problem with nonlinear and nonlocal diffusion coefficient depending on some parameter  $r$ .

We will discuss the existence of branches of solutions parametrized by  $r$ . Pointwise estimates of radial symmetric solutions will also be given.

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### Nonlinear heat equation with a fractional Laplacian in a ball

**Vladimir Varlamov**

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Nonlinear heat equation with a fractional power of Laplacian is considered in a unit ball. Homogeneous boundary conditions and small initial data are examined. For a certain range of powers of the Laplacian global-in-time mild solutions are constructed in the form of an eigenfunction expansion series. Higher-order long-time asymptotics is computed.

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