

Special Session 2: Pattern Formation in Biology and Ecology: from Interfaces to Meta-solutions

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Dynamics of plant-herbivore models: multistability, bursting and spatial expansion

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Yun Kang, Dieter Armbruster and Yang Kuang

We study the time discrete dynamics of general plant herbivore models. We show that the growth model for the plant strongly determines the structure of the dynamics of the full model. We show that for monotone plant growth the system exhibits bursts of parasites, where the quiet period between bursts is determined by the noise level of the system. Assuming a Ricker dynamics as a prototype for unimodal growth of the plant population we find bistability and crises of a strange attractor. This finding suggest two control strategies: Reducing the population of the parasites under some threshold or increasing the growth rate of the plant leaves. Using a 2 compartment model we study the invasion of a parasite free compartment from an infested compartment in the case of a bistable dynamics.

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Blow-up rates of radial large solutions

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Dr. Julián López-Gómez

In this talk we are going to show the blow-up rate along the boundary of the unique positive large solution in Ω , of the non-linear elliptic equation

$$-\Delta u = \lambda u - f(\text{dist}(x, \partial\Omega))V(u)u,$$

where Ω is a ball or an annulus, $f \in \mathcal{C}[0, \infty)$ is a positive and non-decreasing and $V \in \mathcal{C}[0, \infty) \cap \mathcal{C}^1(0, \infty)$ satisfies $V(0) = 0$, $V'(u) > 0$ for all $u > 0$, that there exists $r = r(V) > 0$ such that

$$V(\gamma u) \geq \gamma^r V(u) \quad \text{for all } \gamma > 1, \quad u > 0, \quad (1)$$

and $V(u) \sim Hu^{p-1}$ as $u \rightarrow \infty$ for some constants $H > 0$ and $p > 1$. In addition, we will show a particular classical case, where condition (1) is satisfied.

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Patterns, stability and collapse for two-dimensional biological swarms

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One of the most fascinating biological phenomena is the self-organization of individual members of a species moving in unison with one another, forming elegant and coherent aggregation patterns. Schools of fish, flocks of birds and swarms of insects arise in response to external stimuli or by direct interaction, and are able to fulfill tasks much more efficiently than single agents. How do these patterns arise? What are their properties? How are individual characteristics linked to collective behaviors? In this talk we discuss various aspects of biological swarming as seen through the eyes of a physicist. In particular, we investigate a non-linear system of self propelled agents that interact via pairwise attractive and repulsive potentials. We are able to predict distinct aggregation morphologies, such as flocks and vortices, and by utilizing statistical mechanics tools, to relate the interaction potential to the collapsing or dispersing behavior of aggregates as the number of constituents increases. We also discuss passage to the continuum and possible applications of this work to the development of artificial swarming teams.

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The analysis of a numerical method for a homogeneous, nonlinear, nonlocal elliptic boundary value problem with numerical experiments

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John R. Cannon

In this work we develop a numerical method for the equation: $-\alpha \left(\int_0^1 u(t) dt \right) u''(x) + [u(x)]^{2n+1} = 0$, $x \in (0, 1)$, $u(0) = a$, $u(1) = b$. We begin by establishing the existence and uniqueness of the solution to the nonlinear auxiliary problem via the Schauder fixed point theorem. We then prove the existence of the solution to the nonlinear auxiliary problem by defining a continuous compact mapping and utilizing a priori estimates. From this analysis, we then prove the existence and uniqueness of the problem defined above via the Brouwer fixed point theorem. Next, we analyze a discretization of the above problem and show that a solution to the nonlinear difference problem exists and is unique, and that the numerical procedure converges with error $\mathcal{O}(h^2)$. We conclude with some examples of the numerical process.

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Stability of traveling wave solutions for a nonlinear reaction diffusion system

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We investigate the nonlinear stability of traveling wave solutions for a monotone reaction diffusion system. The traveling wave is unstable in L^p and C^0 spaces, while in some exponentially weighted Banach spaces, the traveling wave solution is asymptotically stable with a shift.

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A host-pathogen model for the spread of a grass species infected with a smut fungus

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Brendan Kelly, Michael Stein and Vincent Martinez

Plants that exhibit invasive characteristics pose a significant threat to ecosystem biodiversity. One such example is broomsedge, *Andropogon Virginicus* L., a colonizing species of grass native to the eastern U.S. It is well-documented that broomsedge

has become a serious threat to Hawaii's ecosystem, and evidence seems to suggest that a smut fungus, *Sporisorium Ellisii*, could be effective in controlling its spread. This talk will introduce a system of partial differential equations that models the interaction of broomsedge with the smut fungus.

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Large solutions of mixed sublinear/superlinear elliptic equations

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For $f(x, u) = p(x)u^\alpha$, $\alpha > 0$ and $p > 0$, the existence of a positive entire large solution (PELS) to $\Delta u = f(x, u)$ is intimately tied to both α and the function p . For example, if $\alpha > 1$ (i.e., f is superlinear), then a PELS exists if $\int^\infty \max_{|x|=r} p(x) dr < \infty$. On the other hand, if $\alpha \leq 1$ (i.e., f is sublinear), then a PELS exists if $\int^\infty \min_{|x|=r} p(x) dr = \infty$ (and p is asymptotically radial). We consider the case where f is a mix between sublinear and superlinear and discover some rather surprising results.

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Stability of traveling waves with noncritical speeds for double

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This talk is concerned with the asymptotic stability of traveling wave solutions for double degenerate Fisher-type equations. By detailed spectral analysis, each traveling front solution with non-critical speed is proved to be linearly exponentially stable in some exponentially weighted spaces. Further by Evans function method and detailed semi-group estimates, each traveling wave solution with non-critical speed is proved to be locally algebraically stable to perturbations in some appropriate polynomially weighted spaces.

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Metasolutions in superlinear indefinite parabolic problems

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Metasolutions are extensions by infinity to the entire underlying domain of a boundary blow-up solution supported on some subdomain. This talk shows that metasolutions are imperative to describe the dynamics of the positive solutions of large classes of semilinear parabolic problems.

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Structural and functional analogies between patterns in real-time streams of behavior and DNA sequences

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Real-time event-streams in human and animal behavior pose a challenge for pattern detection algorithms no less than the detection of patterns in DNA. In both cases, the existing patterning is mostly hidden to the naked eye. Success thus presupposes adequate hypotheses concerning the inherent structure and the corresponding pattern detection algorithms depend on the adequacy of such hypotheses. The data is a set of time point series within the same interval, each series representing the beginning or end of a particular behavior by some agent. A pattern-type, called a t-pattern and corresponding detection algorithms have been developed and used for the detection of human, animal and neuronal interaction patterns. Further structural types have been derived. Striking structural similarities between genes in DNA sequences and the temporal t-patterns discovered in behavior records was noted before any DNA analyses with the t-pattern algorithm were carried out. It is noted that some functional analogies also seem to exist between these spatial and temporal structures. Detailed accounts of the various aspects mentioned above have been published (see www.hbl.hi.is).

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Propagation failure of traveling fronts in discrete inhomogeneous media

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Tony Humphries and Erik Van Vleck

A crude model for electrical conduction in the nervous system is the spatially discrete Nagumo equation. Employing a piecewise linear approximation of the nonlinearity, one can derive exact solutions of this system such that a portion of the medium for conduction is deteriorated, characteristic of diseases that affect the nervous system. Using Jacobi operator theory, wave-like solutions are constructed for a problem with essentially arbitrary inhomogeneous discrete diffusion, and these solutions directly correspond to monotone traveling wave solutions in the case of homogeneous diffusion. A thorough study of the steady state solutions provides necessary and sufficient conditions for traveling waves to fail to propagate due to inhomogeneities in the medium.

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Gradient estimates for boundary blow-up solutions and applications

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We discuss the importance of gradient estimates for boundary blow-up solutions with reference to two main applications. The first is the extension of the classical symmetry result of Gidas-Nirenberg to blow-up solutions in a ball. The second is the characterization of the optimal control mechanism in a state constraint problem for the Brownian motion which, through dynamic programming principle, yields blow-up solutions for the associated viscous Hamilton-Jacobi equation.

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Traveling wave of Auto-catalytic chemical reactions

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In this talk, I shall report some of the recent results on global dynamics and travelling wave solution to the fundamentally important Auto-catalytic chemical reactions of arbitrary order

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Standing pulse solution of a reaction-diffusion equation of logistic growth

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Chongchun Zeng

The dynamical behavior of a reaction-advection-diffusion equation can be partially described by its convergent direction, where a one-dimensional equation for the average profile of the filament is $u_t = Du_{xx} + xu_x + f(u)$. For the logistic growth function $f(u) = u(1 - u)$, we prove the existence and uniqueness of a standing pulse solution on the real line when $D > 1$, and we also show the standing pulse is globally asymptotically stable. A "traveling wave" with varied velocity is also discussed.

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Uniqueness and blow-up rates of large solutions for elliptic equations $-\Delta u = \lambda u - b(x)h(u)$

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In this talk, we establish the blow-up rates of the large positive solutions of the singular boundary value problem

$$-\Delta u = \lambda u - b(x)h(u); u = \infty, \quad \text{when } x \in \partial\Omega$$

where Ω is a smooth bounded domain in R^N . The functions $b(x)$ and $h(u)$ are non-negative continuous functions satisfying $h(u) \sim Hu^p$ for sufficiently large u with $H > 0$ and $p > 1$. We study two cases: (A) Ω is a ball domain and b is a radially symmetric function on the domain; (B) Ω is a smooth bounded domain and b satisfies some local conditions on each boundary normal sections assumed in Theorem 1.2. The

blow-up rate is explicitly determined by functions b and h . The uniqueness of the large solution is also proved.

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A nonlinear system describing an irreversible process

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We will discuss a system for heat equations with discontinuous coefficients and special boundary (interface) conditions. The interface conditions imposed on this problem are continuous for the function (or "temperature"), $[u]_{\pm}^+ = 0$, discontinuous for its derivatives (or "flux"), $[ku_x]_{\pm}^+ \neq 0$, and $u[ku_x]_{\pm}^+ = 0$. These conditions are related to problems with irreversible process in some applications.

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Approximation theorems for a class of fourth order elliptic equations on Riemannian manifolds

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Gilbert Weinstein

For a class of fourth order elliptic equations defined on compact 4-manifolds, we obtain sharp estimates on the asymptotic behavior of blowup solutions near their blowup points.

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