Special Session 30: Some Problems in Difference Equations: Deterministic and Stochastic. Applications

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Non-exponential stability and decay rates in nonlinear stochastic difference equation with unbounded noise

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We consider stochastic difference equation

$$x_{n+1} = x_n\left(1 - hf(x_n) + \sqrt{h}g(x_n)\xi_{n+1}\right),$$

where functions f and g are nonlinear and bounded, random variables ξ_i are independent and $h > 0$ is a nonrandom parameter.

We establish results on asymptotic stability and instability of the trivial solution $x_n \equiv 0$. We also show, that for some natural choices of the nonlinearities f and g, the rate of decay of x_n is approximately polynomial: we find $\alpha > 0$ such that x_n decays faster than $n^{-\alpha+\varepsilon}$ but slower than $n^{-\alpha-\varepsilon}$ for any $\varepsilon > 0$.

It also turns out that if $g(x)$ decays faster than $f(x)$ as $x \to 0$, the polynomial rate of decay can be established exactly, $x_n n^\alpha \to$ const. On the other hand, if the coefficient by the noise does not decay fast enough, the approximate decay rate is the best possible result.

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On existence and stability of periodic solutions for delay difference equations

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We consider Volterra difference equations with variable delays and coefficients. In the case of periodic parameters, a general approach to existence of periodic solutions is proposed. For a particular model of discrete population dynamics, we prove a sharp condition for the existence of a positive periodic solution. In the non-delay case with constant coefficients, local asymptotic stability for this equation implies global asymptotic stability. In the general case, sufficient stability conditions are deduced and illustrated by examples. Finally, we include some discussion on variable parameters vs. constant coefficients case.

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Investigation of Initial and Boundary Value Problems for Linear Differential-Difference Equations by The Polynomial Quasisolution Method

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Consider the scalar linear differential-difference equation (LDDE) of neutral type

$$\dot{x}(t) + p(t)\dot{x}(t-1) = a(t)x(t-1) + f(t),\ t \in R,\ (*)$$

where $p(t)$, $a(t)$ and $f(t)$ are polynomials. When $p(t) = p_0$ and $a(t) = a_0$ are constants and $f(t) \equiv 0$ or has a special representation, the partial analytical solutions for equation $(*)$ can be obtained by Euler's method. Any results related to analytical solutions of equation $(*)$ (where $p(t)$, $a(t)$ or $f(t)$ are functions of t) are unknown to us. When investigating initial and boundary value problems for equation $(*)$, the polynomial quasisolutions method ([1], [2]) has been applied. The basic idea of this method is grounded on the representation of the unknown function in the form of polynomial $x(t) = \sum_{n=0}^{N} x_n t^n$. As a result of substitution of this function into equation $(*)$, there appears a residual $\Delta(t) = O(t^N)$, for which an exact analytical representation has been obtained. In turn, this allows one to find the unknown coefficients x_n and consequently the polynomial quasisolution $x(t)$. A close relationship between the correctness of statement of problems under scrutiny and the model equations with constant coefficients, whose structure of solution is defined by the roots of the characteristic quasi-polynomial, is emphasized.

Several examples are considered.

Reference

[1]. Cherepennikov V. B. Analytic solutions of some functional differential equations of linear systems // Nonlinear Analysis, Theory, Methods & Applications, Vol. 30, т5, 1997.– P. 2641 – 2651.

[2] V.B.Cherepennikov. Polynomial quasisolutions of linear systems of differential-difference equations // Izv. VUZOV. Mathematics, 1999, N10 – P.49-58 (in Russian).

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Stability properties and existence of almost periodic solutions of Volterra difference equations

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In this talk, we shall consider the relationship between the weakly uniformly asymptotic stability and the existence of almost periodic solutions for Volterra difference equations, and then, some applications in population Volterra models are presented to illustrate our main results.

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WKB estimates for 2 x 2 Linear Dynamic Systems on Time Scales

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We establish WKB estimates for 2 x 2 linear dynamic systems with small parameter on a time scale unifying continuous and discrete WKB method. We introduce an adiabatic invariant for 2 x 2 dynamic systems on a time scale, which is a generalization of the adiabatic invariant of Lorentz's pendulum. As an application we present a simple proof of the well known result for continuous time scale and analytic case, that the change of the adiabatic invariant is approaching zero as the small parameter approaches zero. We show that this result is true for the discrete scale only for appropriate choice of graininess depending on the small parameter. In our result the change of adiabatic invariant is approaching zero with polynomial speed because we don't assume any analytic structure. The proof is based on truncation of WKB series and WKB estimates.

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Sampling-Reconstruction Procedure of Markov continuous processes formed by stochastic differential equations of the first order

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There is author's review of some results in the statistical description of the Sampling-Reconstruction Procedure (SRP) of continuous Markov processes. Mathematically these processes are described by the stochastic differential equations of the first order. There are various stochastic differential equations under consideration: 1) linear with constant coefficients; 2) linear with varying in time coefficients; 3) non linear coefficients. In the first variant we deal with the Gaussian process with usual transfer and stationary regimes; in the second variant we have the non stationary Gaussian process with the changed in time parameters; the third variant is concerned with some non-Gaussian processes. On the basis of the solution of the stochastic differential equations one can find the covariance functions and the probability density functions of sampled processes. In result the reconstruction functions and the error reconstruction functions are obtained for the extrapolation and interpolation procedures in the description of SRP. We investigate two types of sampling: the deterministic points of samples and points of samples with jitter. The results about the SRP with jitter of the processes 2) and 3) are unpublished.

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Discretising a polynomial differential equation with fading stochastic perturbation.

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All solutions of a particular stochastic differential equation with polynomial drift and fading, state-independent noise display a.s. asymptotic stability. We examine the uniform Euler-Maruyama discretisation, the dynamics of which are more complex, and less certain.

The set of initial values of the resultant difference equation can be partitioned into a stability re-

gion, an instability region, and a region of unknown dynamics that is "small". These dynamics can only be said to hold with probability at least $1-\gamma$, a value corresponding to the statistical notion of a confidence level, and this description is therefore somewhat fuzzy.

In fact, there is a clear relationship between the specified confidence level, the stepsize parameter, and the size of the stability region. The behaviour of any particular solution of the difference equation can be made consistent with the corresponding solution of the differential equation, for a particular confidence level, by choosing the stepsize parameter sufficiently small.

This behaviour is not observed in a direct simulation of the difference equation on a finite-state machine, using pseudo-random Gaussian numbers. A closer examination reveals that the rate of decay of the simulated stochastic perturbation is slow, making a meaningful numerical investigation of stability difficult.

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Results in Difference Equations of Biological Migration and Immigration Systems.

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We present in this talk some convergence and stability results in biological migration and immigration dynamical systems. Difference equations are involved in these dynamical systems to describe the change in the probability distributions of multivariate characters in subdivided populations with inter-migration. These populations evolve under local selection and migration factors. Both Hard and soft selection types are considered in the study. Using Chebychev rearrangement inequality, see e. g. Karlin(1968) , we highlight a case where the selective force combined with the migration effect leads to an increasing population mean fitness; this exemplifies Fisher's (1930) Fundamental Theorem for Natural Selection. Finally, we prove the stability of the fixed points of Loreau and Mouquet's (1999) dynamical system for the maintenance of local species diversity.

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Stochastic inclusons with noncontinuous multivalued mappings

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We present an existence theorem for stochastic integral inclusion with noncontinuous multivalued integrands. We use the method of lower and upper solutions. Such approach plays a significant and important role in the investigation of existence results for deterministic differential equations or inclusions with noncontinuous but increasing right side. As far as we know, such approach in the case of stochastic inclusions seems to be new. In general, investigating stochastic controlled dynamic systems by methods of multivalued analysis requires an appropriate kind of regularity of their multivalued structure. The properties of Lipschitz continuity, lower or upper semicontinuity and maximal monotonicity are most often considered. We focus on different classes of set-valued functions, namely increasing and "upper separated". Both of them need not satisfy any of classical continuity properties. The upper separatity of a set-valued function is necessary and sufficient for the existence of its convex selection. As a consequence, we deduce the existence of solutions to stochastic inclusions with right sides taken from these classes of multifunctions. Our technique involves combining selection procedures, the method of upper and lower solutions, stochastic comparison and fixed point theorems.

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On positivity and boundedness of solutions of nonlinear stochastic difference equations

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We consider stochastic nonlinear difference equation

$$X(n+1)=X(n)+hf(X(n))+\sqrt{h}g(X(n))\xi_{n+1},$$
$$n\in\mathbb{N},\ X(0)=\varsigma\in\mathbb{R},$$

where $\{\xi_n\}_{n\in\mathbf{N}}$ are independent $\mathcal{N}(0,1)$-distributed random variables, $h>0$. Equation (1) can be viewed as a Euler-Maruyama disretization of stochastic Itô differential equation.

We discuss the following questions: if for all $t\geq 0$ solution X_t of the corresponding stochastic Itô differential equation is positive, or $X_t\in[0,K]$ for some $K>0$, does solution $X(n)$ of the discretization (1) posses the same properties with large probability? In general, the answer is no. However in many cases we are able to discretise the Itô equation according to (1) over a compact interval $[0, T]$ in such a way that corresponding behavior is observed with an arbitrarily high probability.

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Dynamically Consistent Discrete Lotka-Volterra Competition Models

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Sufficient conditions are given for nonstandard finite difference (NSFD) schemes of Lotka-Volterra competition models to be dynamically consistent. The discrete competition models derived from the NSFD schemes preserve the positivity of solutions, boundedness of solutions, local stability conditions, and monotonicity of the continuous Lotka-Volterra system. In other words, we are able to construct discrete-time competition models that behave just like the Lotka-Volterra competition models.

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Stability operators of numerical methods for stochastic differential equations

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The concept and related issues of stability operators of numerical methods for systems of ordinary stochastic differential equations are discussed. The positivity and monotonicity of those operators is exploited to derive stability and boundedness. The example of stochastic theta-methods explains the results and their priority compared to socalled "higher order" methods.

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On some mixing properties of ARCH and time-varying ARCH processes

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We derive strong and two mixing bounds for certain types of ARCH type processes, including time-varying ARCH and ARCH(∞) models.

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