Special Session 31: Nonlinear Waves and Solitons

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Perturbation of topological solitons due to sine-Gordon equation and its type

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The adiabatic dynamics of topological solitons is studied. The solitons due to sine-Gordon equation, double sine-Gordon equation, sine-cosine-Gordon equation and double sine-cosine-Gordon equation are studied. The adiabatic variation of soliton velocity is obtained by soliton perturbation theory.

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Using ocean acoustics to select parameters for large shallow-water soliton simulations

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Ocean solitons can affect the magnitude and phase of acoustic signals that travel through them. Computer simulations have been used to predict the large-scale effects on the acoustic signal. The typical sequence requires that a nonlinear, nonhydrostatic, primitive equation ocean model be initialized by tidal velocity and used to estimate the changes in the environmental parameters due to soliton creation and propagation. These changes in the environmental parameters are used to calculate the related changes in the ocean sound speed field. The last step is to run an ocean acoustic computer model to simulate the propagation of the acoustic signal through the estimated sound speed field and predict the changes in the acoustic signal. Often, the tidal velocity is not precisely known. Variations in the tidal velocity require that the time consuming sequence of computer simulations be repeated. Recently, we have demonstrated a way of estimating the soliton structure that could significantly affect the acoustic signal. This estimation is made before any ocean model simulation is made. This can greatly reduce the number of computer simulations since only ocean model simulations are made for those conditions that might significantly affect the acoustic signal. [Work supported by NRL; PE 62435N]

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The coarse-grain description of interacting sine-Gordon solitons with varying widths

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Building upon our earlier work [1], we study the dynamics of the sine-Gordon equation's kink soliton solutions under the coarse-grain description via two "collective variables": the position of the "center" of a soliton and its characteristic width ("size"). Integral expressions for the interaction potential and the quasi-particles' cross-masses are derived. However, these cannot be evaluated in closed form when the solitons have varying widths, so we develop a perturbation approach with the square of the velocity of the faster soliton as the small parameter. This enables us to derive a system of four second-order ODEs, one for each collective variable. The resulting initial-value problem is solved by standard numerical algorithms to obtain the complete coarse-grain description of the interaction. Then, by comparing the analytical two-soliton solution to the approximate one consisting of the superposition of two one-soliton profiles following the computed trajectories, we show that the coarse-grain description of the dynamics is excellent. Moreover, we demonstrate that, even though it appears the solitons pass through each other, the quasi-particles actually "exchange" their sizes during a collision, thus they scatter each other.

[1] Ivan Christov and C.I. Christov, "Physical dynamics of quasi-particles in nonlinear wave equations," Physics Letters A, doi:10.1016/j.physleta.2007.08.038

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A Perturbation Method for Stationary Propagating Solitons of the 2D Boussinesq Equation

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While the 1D solitons of the so-called 'Proper Boussinesq Equation' (PBE) are exhaustively investigated, very little is known about its 2D localized solutions. Since the solitary waves of PBE exists for sub critical phase speeds $c < 1$, a natural small parameter, $\varepsilon = c^2$, arises in the problem. In this work we expand the solution of the 2D PBE into series with respect to the small parameter, ε, and derive equations for the order $O(\varepsilon^2)$ including. Using the special symmetry of the solution we reduce the problem to six boundary-value problems for ODE's. We solve the latter numerically. compare our results to the available direct 2D finite-difference and spectral numerical solutions, and find the agreement to be very satisfactory. We find that the radially-symmetric profile for $c = 0$ decays slightly super-exponentially as $\exp(-r)r^{-0.5}$, but abruptly changes to algebraic decay r^{-2} for any $c \neq 0$. This kind of 'non-robustness' of the profile as function of the parameter c, is a novel result which is important for understanding the interaction of the localized solutions (quasi-particles) of PBE in multidimension.

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A Study of the Long-Time Solutions of Rosen's Equation Perturbed by a Harmonic Signal on the Boundary

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In this work, we investigate the long-time solutions of Rosen's equation of gas dynamics combustion theory; that is, we will investigate the solutions of Rosen's equation after any transient effects have died down.

Rosen's equation describes the evolution of a finite amplitude pressure wave in a region of uniform, homogeneous gaseous combustion. In particular, we will study Rosen's equation over a region in which homogeneous gaseous combustion satisfies the following three conditions:

1. The gas flow and corresponding pressure waves depend only on a single spatial dimension and time. 2. The effects of viscosity in the longitudinal direction can be neglected. 3. The surplus local burning rate can be expressed as a parabolic function of the local pressure.

With these conditions, Rosen's equation takes the form

$$\frac{\partial^2 P}{\partial t^2} - [2b(P - P_c) + a]\frac{\partial P}{\partial t} - \frac{\partial^2 P}{\partial x^2} = 0 \qquad (1)$$

,

where $P(g/cm^3)$ is the pressure of the gas, and the constants P_c (the pressure of the vessel), $a < 0$, and $b \neq 0$, determine when the surplus local burning rate is physically realizable.

The investigation will be carried out by considering a harmonic signal that perturbs the initial state of a homogeneous gas in a cavity, such that conditions (1)-(3) above are satisfied, triggering combustion, and then studying the development of the pressure after all transient effects have dissipated.

Formally, we will perform a perturbation analysis on Rosen's equation using the following boundary conditions:

$$P(0,t) = P_c e^{-i\omega t} \qquad (2)$$

$$P(\infty, t) = 0 \qquad (3)$$

The quantity $2b$ will be used as the perturbation parameter for the analysis. Dispersion relations and phase speeds for each perturbation will be obtained.

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Nonlinear Phenomena in Acoustics: Traveling Waves, Bifurcations, and Singular Surfaces

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Traveling wave solutions (TWS)s are explored in the context of nonlinear acoustics. Exact expressions are given, including one involving the recently introduced Lambert W-function, along with asymptotic and stability results. Poroacoustic propagation under Darcy's and Forchheimer's laws is examined, as well as acoustic phenomena in thermoviscous fluids. Additionally, a connection between discontinuity formation in the TWS and results from singular surface theory is established. Lastly, if time permits, applications to nonlinear kinematic wave phenomena (e.g., second-sound and traffic flow) are briefly noted. [Work supported by ONR/NRL funding (PE 061153N).]

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Characteristic line scheme for wave solutions of hierarchical size-structured model with nonlinear growth, mortality and reproduction rate

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In the present work we propose a new scheme based on characteristic lines for solving hierarchically structured population model with nonlinear growth, mortality and reproduction rate. The scheme is stable and have second order of approximation in x and t. The idea of the method is not to follow the characteristics from the initial condition, but for each time-step to recover trough the characteristics the position at the previous time level of the functional values that arrive at a grid point on the net tile level. Numerical results confirm second order of convergence of the new schemes. The scheme is validated for two exact solution: a continuous and a discontinuous. In addition, we compare the results of the new scheme with two known numerical numerical schemes for the model under consideration.

The new scheme is very efficient for investigating the propagation of disturbances (waves) in size-structured model with nonlinear growth, mortality and reproduction rate. We have been able to get accurate prediction for the propagation of a disturbance from the boundary inside the region for the case of strongly nonlinear coefficients.

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A NSFD Scheme for the Drift-Diffusion System

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We develop a nonstandard finite difference (NSFD) scheme to obtain numerical solutions for the set of PDE's governing drift/diffusion of electrons and holes in a semiconductor. The first two PDE's are nonlinear advection, diffusion equations and model the behavior of the electrons and holes. The third equation involves the electric field and can be reformulated as a Laplace equation in terms of the potential (the electric field can then be calculated by taking the gradient of the potential). Our NSFD scheme incorporates a positivity condition for the electron and hole densities, and thus, unlike standard finite difference methods, resolves several stability issues. We will present the derivation of this scheme and show its application to an actual experimental situation.

This research was supported in part by DOE. We thank Ryan Murdick (Department of Physics and Astronomy, Michigan State University) for alerting us to this problem and providing useful experimental data that allowed us to construct a valid set of NSFD schemes.

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Numerical simulation of internal waves generated by local density perturbation in stable stratified fluid

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The dynamics of local density perturbation in a surrounding ambient stably stratified fluid is examined by numerical simulation based on the Eulear and Navier-Stokes equations in the Oberbeck-Boussinesq approximation. Numerical algorithms are constructed with using both Eulear-Lagrangian and Eulear coordinate systems and methods of splitting. The computational results demonstrate the substantial effect of the fluid density distribution on the internal wave pattern generated by the collapse of a local density perturbation. In the case of local density perturbation dynamics in a narrow pycnocline in an inviscid fluid, the stationary internal solitary waves are generated. The propagation velocity of the main wave and its amplitude satisfy the known asymptotic Benjamin relation. The numerical experiments were carried out to estimate the viscosity effect on the generation and propagation of internal waves in a pycnocline. We suggested a simple algebraic dependence relating the peak amplitude of a wave to its velocity in a viscous fluid in a "narrow" pycnocline: with one of the values known from the calculations, the second one is found from the above Benjamin relation.

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Experiments, Simulations and Nonlinear Dynamics Modeling of Galton's Board

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An investigation of the motion of a single particle rolling down a Galton's board is carried out using experiments, simulations and discrete dynamical models. The results from the experiment, simulation and discrete dynamical system approaches with regard to such properties as lateral diffusion coefficients and end distributions are found to be in reasonably good agreement. Furthermore, the existence of chaotic regimes is indicated by the experiments and simulations, and proven using the simple dynamical models for certain ranges of the key system parameters.

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Repulsive Soliton Collision in Strongly Coupled Nonlinear Schrödinger Equations

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Abstract: The strongly coupled nonlinear Schrödinger equations are studied numerically to gain insight into an increasingly complex family of collision interactions. Specifically, the case of apparent repulsion between soliton solutions are considered in terms of phase velocity, relative shift, and apparent trajectory. Both elastic and inelastic collisions are discussed.

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anisotropic effects on poroacoustic waves

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Nonlinear acceleration waves are considered in porous media. Our goal is to assess the effect of anisotropy on poroacoustic waves. We show anisotropy may have a pronounced effect on sound attenuation. We consider this aspect in the case of "designer" porous materials which are being produced with regard to sound and heat insulation.

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Parallel Numerical Methods for Solving Nonlinear Evolution Equations

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Nonlinear evolution equations of the NLS, complex MKdV and KdV types are of tremendous interest in both theory and applications. In this talk we introduce parallel algorithms for numerical simulations of these types of equations. The parallel methods are implemented on the Origin 2000 and the IBM p655 multiprocessor computers. Our numerical experiments have shown that these methods give accurate results and considerable speedup.

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Impact of the Initial Polarization on the Collision Dynamics of Quasi-Particles Governed by Vector NSE

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In this work, the system of Coupled Nonlinear Schrodinger Equations (CNLSE) is solved by a conservative difference scheme in complex arithmetic developed in earlier author's work. The initial condition represents a superposition of two one-soliton solutions of different (generally elliptic) polarization. This requires a preliminary solution of a system of nonlinear ODE's for the envelop amplitudes which are not sech-es as in the case of linear and circular polarizations.

The interaction (collision) of the solitons and their quasi-particle (QP) behaviour is thoroughly examined for different configurations of the initial system of QPs. The effect of the nonlinear coupling on the QP dynamics is elucidated. We find that depending on the value of the cross-modulation parameter, the carrier frequency of a QP can change after a collision with another QP, i.e., the polarization ratio changes. It is found out that the elliptically polarized solitons do exist even for those values of the carrier frequency and phase velocity, for which the corresponding linear polarized sech-solutions do not exist. Data about the phase speeds, energies, momenta and masses of the QPs after the collision are compiled and discussed.

The effects found in the present work seem to be novel and enrich the knowledge about the intimate mechanisms of interaction of polarized QPs.

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