Special Session 34: Wave Propagation in Nonlinear Materials

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Signal transmission in multichannel optical fiber communication systems.

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We study the combined effect of delayed Raman response and bit pattern randomness on pulse propagation in massive multichannel optical fiber communication systems. The propagation is described by a perturbed stochastic nonlinear Schrödinger equation, which takes into account changes in pulse amplitude and frequency as well as emission of continuous radiation. We perform extensive numerical simulations with the model, and analyze the dynamics of the frequency moments, the bit-error-rate, and the mutual distribution of amplitude and position. The results of our numerical simulations are in good agreement with theoretical predictions based on the adiabatic perturbation approach.

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Nonlinear stress & strain wave communication in biological soft matter

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We revisit a classical topic: response functions of viscoelastic layers in large amplitude oscillatory shear. Motivated by questions concerning biological layers, we focus on extreme values in boundary stress signals in a parallel plate geometry, and their dependence on material properties, layer thickness, or frequency of imposed strain. We identify a robust oscillatory structure in boundary stress signals, presented in terms of a layer height sweep and then generalized to scaling behavior with respect to all experimental parameters. The structure indicates redundant mechanisms for stress communication (by tuning to the peaks) and stress filtering (by tuning to the valleys). Analysis on the simplest nonlinear model, the upper convected Maxwell model, is followed by a Giesekus model simulation of coupled nonlinear partial differential equations.

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Fractal Structure in Solitary Wave Interactions

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The following scenario has been seen in many non-integrable, dispersive, nonlinear PDE over the last 25 years: two solitary waves are propagated on a collision course. Above some critical velocity v_c, they simply bounce off each other. Below v_c they may be captured and merge into a single localized mass, or they may interact a finite number of times before escaping each other's embrace. Whether they are captured, and how many times the solitary waves interact before escape, depends on the initial velocity in a complicated manner, often remarked, though never shown, to be a fractal (a chaotic scattering process). This has been observed in coupled NLS, sine-Gordon, ϕ^4, and others.

These PDE systems are commonly studied by (nonrigorously) deriving a reduced set of ODE that numerically reproduce this behavior. Using matched asymptotics and Melnikov integrals, we give asymptotic formulas for vc and for certain salient features of the fractal structure. We derive a discrete-time iterated map through which the entire structure can be unravelled. Recent results focus on how radiation-induced dissipation effects the fractal structure.

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Solitary Waves in Discrete Media in the presence of Four-wave mixing products

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In this talk, I will discuss solutions that arise in a vector discrete model of the Nonlinear Schrödinger equation where nonlinear inter-component coupling and four-wave mixing are taken into account. We show that the solutions to this model give rise to two single mode branch solutions as well as two mixed mode branch solutions. These solutions are obtained explicitly and their stability is analyzed in the so-called anti-continuum limit. Also, we connect this analysis to recent experiments that motivated this work.

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Localized traveling pulses in the Swift-Hohenberg equation with and without broken reflection symmetry

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The Swift-Hohenberg equation is a reversible partial differential equation describing pattern-formation in one spatial dimension. In the bistable case the equation is known to admit a large multiplicity of steady spatially localized states in a parameter region known as the pinning or snaking region. The structure of this region is a consequence of the spatial reversibility of the equation. In this talk I will describe the unfolding of this structure when the reversibility is weakly broken, and the steady states turn into spatially localized traveling pulses. Some of these pulses remain as traveling states even when the reversibility symmetry is restored.

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Nonlocal stabilization of localized solutions to damped-dispersive equations

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A variety of physical phenomena give rise to nonlocal coupling of optical fields, such as when light propagates through media subject to heating or photorefractive damage. The nonlocal terms in the resulting evolution equations can stabilize localized solutions, in particular solitons and traveling waves, that are otherwise unstable and therefore physically irrelevant. We describe this stabilization geometrically and through a collective coordinate reduction in a system subject to homoclinic snaking that is derived from an optical cavity containing a thermally driven parametric gain medium.

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Optical Solitons in PT Periodic Potentials

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We investigate the effect of nonlinearity on beam dynamics in parity-time (PT) symmetric potentials. We show that a novel class of one- and two-dimensional nonlinear self-trapped modes can exist in optical PT synthetic lattices. These solitons are shown to be stable over a wide range of potential parameters. The transverse power flow within these complex solitons is also examined.

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Coarse-graining noise in nonlinear systems with scale-separation

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I will discuss three different methods of coarse-graining noise in nonlinear systems. The first method is using random solvability conditions on the level of stochastic differential equations. The second method employs a multiple-scale expansion of the associated Fokker-Planck equation and the third method is based on scale-separation in path integrals kernels. Following the theoretical discussion of the methods I will present two applications: (a) a characterization of the impact of noise on dispersion-managed solitons and (b) the derivation of a stochastic equation describing the evolution of ultra-short pulses in nonlinear media.

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Recent developments on rogue waves

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In this talk we will discuss physical and statistical properties of rogue wave generation in deep water from the perspective of the focusing Nonlinear

Schrödinger equation and some of its higher order generalizations. Numerical investigations and analytical arguments based on the inverse spectral theory of the underlying integrable model, perturbation analysis, and statistical methods provide a coherent picture of rogue waves associated with nonlinear focusing events. Homoclinic orbits of unstable solutions of the underlying integrable model are certainly candidates for extreme waves, however for more realistic models such as the modified Dysthe equation two novel features emerge: (a) a chaotic sea state appears to be an important mechanism for both generation and increased likelihood of rogue waves; (b) the extreme waves intermittently emerging from the chaotic background can be correlated with the homoclinic orbits characterized by maximal coalescence of their spatial modes.

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Dynamics of sheared polymer-particulate nanocomposites

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Qi Wang

We will present a new kinetic theory for the polymer-nanocomposite flows to study the material properties of the industrial important novel material. A closure model for weakly semiflexible nanorods and platelets is derived from the kinetic theory. The flowing material under shear is investigated in both dilute and semidilute regimes. Phase behavior and the corresponding rheological behavior will be discussed in details.

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Phase-Field Models for Biofilm Growth, Expansion, and Biofilm-Flow Interaction

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We derive a set of phase field models for biofilms using the one-fluid two-component formulation in which the combination of extracellular polymeric substances (EPS) and the bacteria are effectively modeled as one fluid component while the collective ensemble of nutrient and the solvent are modeled as the other. The biofilm is assumed an incompressible continuum. Two growth modes are identified in linearized analysis. Numerical simulations are carried out in one and two space dimension using a velocity-corrected projection method for incompressible flows. Biofilm growth, expansion, streaming, rippling, and detachment are simulated in shear cells numerically. Viscoelastic properties of the biofilm is investigated as well.

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Structures of nematic polymers in the flow

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We study the effect of the flow on structure formations of nematic polymers. Numerical simulations are based on the molecular kinetic theory. Various attractors and phase transitions are investigated. Rheological properties (alignment properties, order parameters, normal stress differences, shear stress, etc.) for each attractor are then reported.

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