

## Special Session 35: Differential Equations of Mixed Type Arising in Engineering, Biology and Ecology

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### A Reaction-Diffusion Model of Bovine Viral Diarrhea Virus (BVDV) Infection

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Bovine Viral Diarrhea Virus (BVDV) Infection is believed to be widespread in the world national herd. The infection tends to be endemic in many populations, reaching a maximum level of 1-2% of the cattle being persistently infected (PI) and 60-85% of the cattle being antibody positive. The majority of acute BVDV infections are caused by non-cytopathic viruses. Animals acutely infected with BVDV undergo a transient alteration of the absolute number of circulating leukocytes. More specifically, BVDV can infect and replicate in macrophages, and they can alter the functions and cause the depletion of B and T lymphocytes. Since the density of each species is not homogeneously distributed, partial differential equation (PDE) modeling is worth pursuing. In this paper, a reaction-diffusion system of four equations that model the underlying dynamics of the interaction between the basic immune cells and the virus is presented, numerically solved, and analyzed. Also, we explore the pathogenesis of more severe clinical outcomes of BVDV and the role of secondary infection by other pathogenesis. Finally, the mathematical results will be used to give some recommendations that might help the medical community in the controllability of the disease

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### Modeling Predator-Prey Interactions

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Corpus Christi Bay, Corpus Christi TX, is a desirable location to study predator-prey interactions. The bay is surrounded on three sides, by three different environments. Heavy industries, such as refining

and manufacturing are found to the north, residential and military establishments are found to the south, and a mostly undeveloped barrier island is located to the east with a connection to the Gulf of Mexico. The bay and surrounding waterways are an important nursery for marine life. With a growing human population that threatens to impact the waterways in a negative way, it is necessary to understand how these anthropogenic influences will affect the marine ecosystem. If the juvenile organisms at the lower end of the food chain are influenced by changes in their environment these changes will be noticed throughout the food chain. The predator-prey chain in this system begins with phytoplankton and ends with fish, mammals, and birds. A multi-species predator-prey model has been set up, numerically solved, and analyzed. The model is used to study the Corpus Christi bay predator prey interactions and give insight into future scenarios and its impact on surrounding areas.

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### Stability analysis for the bacteria population

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The dispersal of bacteria colonies in a nutrient described by a pair of reaction-diffusion equations was studied in many publications, and the results were confirmed by the experimental work. However, the nutrient was assumed to be non-renewable, and the natural mortality process in the bacteria colonies was neglected, with the natural life expectancy for bacteria being much higher than the characteristic time until the nutrient expired. We study the model when the nutrient is renewable (either reproduces or is supported artificially). We study linear stability of each of the three possible spatially uniform equilibria: total extinction, bacteria-free model and co-existence equilibrium. From the simplest case, when Type I of response is assumed (or, as in many cases, Type II response is approximated by Type I), we proceed to the analysis of a model with Type II response, where for various values of parameters both monotone and

oscillatory instability could be observed.

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### **Instability analysis on spatial vector solitons via a local min-orthogonal method**

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In nonlinear optics, spatial vector solitons are "particle-like" unstable nonlinear objects, corresponding also to co-existing excited states. Physically, they may be stable enough for experimental observations; some of them are even very robust and can survive a wide range of perturbations. Mathematically, however, they are saddle points of certain dual functionals. In this talk we study the instability analysis on multiple saddle points (especially multiple co-existing saddle points) and its applications in computing both 2- and 3-component spatial vector solitons which exist and propagate in a saturable nonlinear medium. Based on a local min-orthogonal method developed, we present some estimates of the Morse index of saddle points. Finally, numerical results are also given.

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### **Some spatial soliton solutions of two-dimensional generalized nonlinear Schrödinger equation with variable coefficients**

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NLSE (the nonlinear Schrödinger equation) is one of the most important universal nonlinear models arising naturally in many physical systems such as nonlinear optics, condensed matter physics, Bose-Einstein condensates, plasma physics, and hydrodynamics. In this talk, we look at a certain NLSE modeling the propagation of 2D spatial solitons in bulk optical media with variable coefficients. By using an F-expansion techniques, we construct a class of exact periodic wave solutions for the NLSE with distributed dispersion, nonlinearity, and gain or loss. In the limiting cases of certain parameters, those periodic wave solutions acquire the form of localized special soliton solutions. Graphics will be illustrated.

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### **Mathematical Analysis for a Model Arising From Public Goods Games**

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**Xiaojie Hou and Xin Lu**

We study the effects of the altruistic behaviors in a public goods game model, which describes the competition between the farmers and the exploiters. Corresponding to different parametric regions, we analyze in details the stability of the equilibrium states and obtain attraction regions for stable equilibria. Then using the upper-lower solution method and monotone iterations, we further show that for a family of wave speeds, there exist traveling wave solutions connecting one of the unstable states to the stable state. This answers a conjecture made by Wakano. The results indicate that when the penalty for the altruistic behavior is small, the growth rate of the population determines its survival or extinction states in the long run. Furthermore, if the two populations have the same total growth rate, altruism in the competition leads to a wide range of co-existent states. Numerical simulations are also presented to illustrate the theoretical results.

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### **The singular perturbation analysis in the modeling of fracture using a new multiscale theory**

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**Jay R. Walton, John C. Slattery and Tsvetanka Sendova**

In the effort of coupling classical continuum mechanics with quantum mechanical calculations, we encountered a nonlinear boundary value problem from a novel approach to modeling fracture of a brittle material. Our approach leads to an interesting analogy with boundary layer theory. Within the interfacial region the effects of the intermolecular forces must be preserved. Outside the interfacial region (about 100nm beyond the phase interface) the effects of intermolecular forces are neglected. As people did in boundary layer theory, we used singular perturbation method and obtained interesting results. Because of the complicate physical features of the problem, there are multiple perturbation parameters, which needs

more research work. This could serve as a demonstration of the new use of an old mathematical tool but with advanced features.

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### **Understanding and Control Chaos in Nonconvex/Neoconservative Dynamical Systems:**

**David Gao**

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In this talk, the speaker will present a potentially powerful canonical dual transformation method and the associated triality theory for solving a large class of nonconvex dynamic problems. He will first show that by using this method, a class of nonlinear dynamical problems can be transformed into the so-called DAE (differential-algebraic equation) system. The chaotic trajectories in phase space form an invariant set in dual phase space. The interesting triality theory discovered recently can be used to control the chaotic behavior of the nonconvex systems, and to identify both global minimizer and local extrema. Based on this triality theory, a powerful primal-dual algorithm is suggested. Applications will be illustrated by an interesting example on dual feedback control against chaotic vibration of a large deformation beam model proposed by the speaker. Some interesting new phenomena, i.e. the so-called meta-chaos and tri-chaos will be illustrated by movie.

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### **Spectral analysis of stationary solutions of the Cahn-Hilliard equation**

**Peter Howard**

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I will discuss recent results on the stability of stationary solutions for the Cahn-Hilliard equation on  $\mathbb{R}^d$ ,  $d \geq 1$ . For the case  $d = 1$  there are precisely three types of bounded non-constant stationary solutions: periodic solutions, pulse-type reversal solutions, and monotonic transition front solutions. The periodic and reversal solutions are both spectrally unstable, while the transition front solutions are spectrally and nonlinearly (phase-asymptotically) stable. The cases  $d \geq 2$  are more complicated, and I will discuss the types of stationary solutions that arise in these cases and what is known about the stability of such

solutions.

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### **Synchronization of Chaotic Systems using intermittent linear feedback control**

**Tingwen Huang**

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**Chuangdong Li**

This paper studies the synchronization of chaotic systems by means of intermittent feedback method. A sufficient synchronization criterion for a general intermittent linear state error feedback control is obtained by using Lyapunov function and differential inequality method. Numerical simulations on the chaotic Chua oscillator are presented to verify the theoretical results.

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### **A framework of 4D-Var and its mathematical analysis**

**Lijian Jiang**

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**Craig C. Douglas**

In this talk, we give a unified analysis and a general procedure for 4D variational data assimilation (4D-Var). In functional partial differential equation setting, the adjoint equation method, sensitivity analysis, and multicomponent operator splitting are discussed. Nonlinear optimization methods and convergence analysis are also investigated for the 4D-Var.

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### **Stability analysis of the inhomogeneous equilibrium for axially and transversely excited non-linear beam**

**Emine Kaya**

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**E.Aulisa, A.Ibragimov and P.Seshaiyer**

In this work we consider the dynamical response of a non-linear beam with viscous damping perturbed in both the transverse and axial directions. The system is modeled using coupled non-linear momentum equations for the axial and transverse displacements. In particular we show that for a class of boundary

conditions (beam clamped at the extremes) and uniformly distributed load, there exists a non-uniform equilibrium state. Two model of damping are considered: first, third and fifth order dissipation terms (in each the first derivative is with respect to time, the others with respect to the axial coordinate). We show that the excited beam is stable near the equilibrium for any perturbation in the presence of any order dumping term. In addition we observe that if the damping coefficient vanishes in time with the order of  $1/\text{time}$ , then the system can become unstable. An energy estimate approach is used to identify the appropriate space, in which the solution of the perturbed system is stable.

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### **Nonlinear interfacial dynamics and mechanisms of liquid transport in a gas-liquid core-annular flow**

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**Roberto Camassa and Michael A. Stoellinger**

A two-phase core-annular flow in a cylindrical pipe is considered. The inner core is assumed to be a strongly pressure-driven gas flow. The other phase is highly viscous fluid lining the inner wall of the pipe. We present a two-phase gas-fluid model to predict the mean thickness of the liquid layer in the experiment by Kim et al., J. Appl. Physiol. 60(3):908-917, 1986, where a given fixed gas-flow rate drags the liquid injected into the pipe at a fixed feed rate. The high-Reynolds gas flow in the experiment is evidently turbulent. We introduce a simple turbulence model to represent the mean flow field of the turbulent gas flow. This mean flow is then incorporated as a forcing term into the equation for the liquid. We derive a nonlinear evolution equation based on the lubrication approximation for the interface and numerically study the interface evolution of an initially axisymmetric disturbance of the annular film of viscous liquid. The mean thickness of the liquid layer in the experiment can be accurately predicted using this model, and the existence of the ring-like waves reported in the experiments is confirmed by the interfacial dynamics of the model.

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### **Spin Dynamics of a Long Nano-Magnet Driven by Electrical Currents**

**Y. charles Li**

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**Yueheng Lan**

I will show several new results on the Spin Dynamics of a Long Nano-Magnet Driven by Electrical Currents. Applications of the problem include enhancing speed and capacity of computer hard drive.

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### **A Robust Finite Element Method for Singularly Perturbed Convection-Diffusion Problems**

**Runchang Lin**

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In this talk, a discontinuous Galerkin method is developed based on a least-squares functional for solving singularly perturbed convection-diffusion problems. It can be shown that the method is well-posed and convergent. Numerical tests are included to illustrate the error bounds.

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### **Developments And Future Trends in the Harmonic Balance Method for Nonlinear Dynamical Systems**

**Liping Liu**

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**Earl H. Dowell and Kenneth C. Hall**

The harmonic balance method is a very efficient and useful approach for characterizing and predicting the response of nonlinear dynamical systems undergoing periodic or quasi-periodic oscillations, whether self-excited or due to external harmonic excitation. Many decades after the first complete presentation of the method, it has been reformulated and further developed by incorporating various extensions and enhancements. The dynamic systems considered range from a simple one degree-of-freedom oscillator such as Duffing's oscillator to complex high dimensional systems with non-analytical nonlinearities in models of fluid-structural interactions. An overview will be presented of the formulation of the basic method and a brief description of the variants. For

each version of the method, significant studies will be highlighted with the emphasis on insights into the accuracy of the proposed methods versus complexity and computational requirements. Subjects will include the variants of the harmonic balance method, the advantages and disadvantages of the several variants, the implementation of the variants in various applications, and possible future developments for incorporating complex nonlinearities in very high dimensional systems.

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### **Spatiotemporal patterns and chaotic burst synchronization in a small-world neuronal network**

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**Yanhong Zheng**

he spatiotemporal patterns and chaotic burst synchronization of a small-world neuronal network are studied in this paper. Synchronization parameter, similarity parameter and order parameter are introduced to investigate the dynamics behaviour of the neurons. Chaotic burst synchronization and nearly complete synchronization can be observed as the link probability and the coupling strength are large enough. It is found that with increasing the link probability and the coupling strength chaotic bursts become appreciably synchronous in space and coherent in time, and the maximal spatiotemporal order appears at some particular values of the probability and the coupling strength, respectively. The larger the size of the network, the smaller the probability and the coupling strength are needed for the network to achieve burst synchronization. Moreover, the bursting activity and the spatiotemporal patterns are robust to small noise.

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### **Coupled Electromechanical Effects in Nanostructures: Electrostriction in the Multidimensional Case**

**Roderick Melnik**

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**Mehrdad Bahrami-Samani and Jean Zu**

In this contribution, we analyze higher order electromechanical effects in low dimensional semiconductor nanostructures, focusing on quantum dots. While the mathematical theory of linear electromechanical interactions was put on a rigorous foundation earlier and efficient numerical procedures were developed [e.g., Appl. Math. Comp., 107, 2000, 27-55], many questions remain, already at the level of model development, in the nonlinear case. Formally, the model is based on a system of PDEs consisting of elasticity equations coupled with the Maxwell equation. In the context of nanostructures, experimental evidence suggests that a number of higher order effects may become important in predicting their properties. Nevertheless, there are no currently models that account for such effects in a consistent manner. Elastostriptive effects have recently been discussed in [PRL, 96, 2006, 187602]. However, no attempts have been made to estimate the significance of electrostrictive effects, in particular when it comes to the modeling of quantum dots where mathematical models should go beyond one-dimensional descriptions, sufficient only for quantum wells and idealized quantum wires. Nonlinear constitutive relationships, coupling mechanical and electric fields in a non-trivial manner, bring major challenges in dealing with this problem. In this contribution, we provide a systematic characterization of nonlinear electromechanical effects in low dimensional semiconductor nanostructures and report several computational results for wurtzite GaN and zinc-blende GaAs quantum dots obtained with the developed models.

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### **Nonlinear Semigroups and the Question of Boundary Conditions for the Tricomi Equation**

**John Neuberger**

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A review of recent generator-resolvent theory for nonlinear semigroups is given. Application is made to the problem of characterizing the set of all solutions to PDEs for which conventional boundary conditions are not known. The theory is illustrated for a Tricomi equation which is elliptic in one part of a region and hyperbolic in another part.

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### **Stability analysis using an energy estimate approach of a reaction-diffusion model of athero-**

genesis

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**A.I. Ibragimov, J. R. Walton and C. J. McNeal**

This paper considers modeling atherogenesis, the initiation of atherosclerosis, as an inflammatory instability. Motivated by the disease paradigm articulated by Russell Ross, atherogenesis is viewed as an inflammatory spiral with positive feedback loop involving key cellular and chemical species interacting and reacting within the intimal layer of muscular arteries. The inflammation is modeled through a system of nonlinear reaction/diffusion/convection partial differential equations. The inflammatory spiral is initiated as an instability from a healthy state which is defined to be an equilibrium state devoid of certain key inflammatory markers. Disease initiation is studied through a linear, asymptotic stability analysis of a healthy equilibrium state. Various theorems are proved giving conditions on system parameters guaranteeing stability of the health state and conditions on system parameters leading to instability.

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### **Numerical Modeling of the Tear Film Rupture Employing Lubrication Theory**

**Asad Salem**

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**Dr. Mufid Abudiab**

The drainage of the precorneal tear film in humans is studied. A fluid dynamic model for the drainage of the aqueous layer is developed that includes rheological effects. The Ostwald der Waals type power law model is employed to model the tear film. The nonlinear evolution equation for the film is formulated using the balance equations including a body force term due to van der Waals molecular attractions, lubrication theory and perturbation expansion method. The governing equation was solved by Spectral Methods technique as part of an initial value problem for spatial periodic boundary conditions. The results indicate that the rheological effects of the tear film fluid affect the film drainage process and therefore be included in models for tear film damage.

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### **A Fully Integrated Finite Element Hydrody-**

### **namic Model for Coastal Regions**

**Hua Shan**

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This talk presents a general framework of an integrated surface, subsurface, and overland hydrodynamic model. The integrated model includes three fundamental components: a three-dimensional surface water flow and transport module, a three-dimensional subsurface flow and transport module, and a two-dimensional overland flow and transport module. In the surface water regime, the moving free surface is explicitly handled by solving the kinematic boundary condition equation and using the moving grid method based on a node-repositioning algorithm to track the deformation of the surface water body. The Arbitrary Lagrangian-Eulerian (ALE) representation is adopted for all transport equations in the system. The numerical solution is obtained using either the finite element (FE) method or the mixed Lagrangian-Eulerian (particle tracking) and FE method. The dynamic coupling of the surface-subsurface, surface-overland, and overland-subsurface modules in the model enables the interactions of flow and transport across the boundaries of the three regimes.

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### **Bifurcations in diffusive predator-prey systems**

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**Sze-Bi Hsu, Junjie Wei and Fengqi Yi**

A diffusive predator-prey system with Holling type-II functional response subject to Neumann boundary condition is considered, and the prey is assumed to be either diffusive or immobile. We carry out Hopf bifurcation analysis and steady state bifurcation analysis to the system. In particular we show the existence of the spatial-dependent periodic solutions and patterned stationary solutions.

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### **A Model of Signaling Pathways in Embryonic *Xenopus laevis***

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**Akif Uzman and Jeong-Mi Yoon**

A mathematical model of biochemical signaling pathways in the embryonic development of *Xenopus laevis* is presented in this talk. The model consists of a system of coupled, nonlinear ordinary differential equations. By using linear stability analysis and bifurcation theory, the properties of the model is described. Numerical computations include bifurcation studies, which may be helpful in elucidating the interactions within the biochemical signaling network in embryonic *Xenopus laevis*.

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### **Inequalities for $C^\alpha$ -norms of Solutions of Fully Nonlinear Elliptic Equations**

**Antonio Vitolo**

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Many problems in physics and engineering lead to linear second order uniformly elliptic equations in nondivergent form

$$Lu := a_{ij}D_{ij}u + b_iD_iu + cu = f$$

in a domain  $\Omega$  of  $R^n$ , where

$$\lambda|\xi|^2 \leq a_{ij}(x)\xi_i\xi_j \leq \Lambda|\xi|^2$$

for all  $x \in \Omega$  and  $\xi \in R^n$  with positive ellipticity constants  $\lambda \leq \Lambda$ .

In a recent paper Li and Nirenberg have generalized Glaeser's unidimensional inequality to positive strong solutions  $u \in W_{loc}^{2,p}(\Omega)$  of uniformly elliptic equations with continuous coefficients  $a_{ij}$ , establishing local  $C^\alpha$ -estimates in the framework of Krylov-Safonov elliptic theory.

We discuss the extension to solutions of fully nonlinear equations

$$F(x, u, Du, D^2u) = f,$$

where  $F$  is uniformly elliptic, namely

$$\lambda Tr(Y) \leq F(x, t, \xi, X+Y) - F(x, t, \xi, X) \leq \Lambda Tr(Y),$$

for all  $Y \geq 0$ . A linearized approach does not work unless  $F$  and  $u$  are sufficiently smooth. To deal with more general situations, such as viscosity solutions of Bellman-Isaacs equations, we employ nonlinear techniques essentially due to Caffarelli. In particular, qualitative Hölder estimates are obtained in very general hypotheses, while explicit gradient estimates, extending some classical results for the Poisson equation, can be shown under suitable invariance assumptions on  $F$ .

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### **Analysis of Time to Event Data With Functional Predictors**

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The problem of interest is to study time to event type of responses with sparse functional predictors. Our semi-parametric methodology utilizes spline bases function such that the function is represented by a spline coefficient vector with small dimension. Application examples are discussed.

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### **Global Attractor of the Schnackenberg Equations**

**Yuncheng You**

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One of the most widely studied reaction-diffusion models for pattern formation in embryogenesis and in some chemical waves is the Schnackenberg equations. In this work, the existence of a global attractor for the solution semiflow of the Schnackenberg equations of space dimension  $n \leq 3$  is proved. An effective decomposition method is exploited to overcome the difficulties in proving the asymptotical compactness due to the nonlinearity that does not satisfy the dissipative sign condition. It is also shown that the Hausdorff dimension and the fractal dimension of the global attractor are finite.

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### **Approximation theorems for a class of fourth order elliptic equations**

**Lei Zhang**

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**Gilbert Weinstein**

For a class of fourth order elliptic equations defined on a four dimensional compact Riemannian manifold, we obtain sharp estimates on the asymptotic behavior of blow-up solutions near their blow-up points.

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