

Special Session 36: Nonlinear Elliptic and Parabolic PDEs with Applications

Junping Shi, , USA

Traveling Waves for System of Nonlocal Evolution Equations with Bistable Nonlinearity

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The existence, uniqueness and stability of traveling wave solutions for system of nonlocal evolution equations with bistable nonlinearity are established. It is also proved that traveling waves are monotone and exponentially asymptotically stable, up to translation.

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Structure of the sets of regular and singular radial solutions for a semilinear elliptic equation

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Eiji Yanagida

This paper is concerned with the structure of the set of radially symmetric solutions for the equation

$$\Delta u + f(u) = 0 \text{ on } \mathbf{R}^n \setminus \{O\},$$

with $n > 2$. Here the nonlinear term f is assumed to be a smooth function of u that is positive for $u > 0$ and is equal to 0 for $u \leq 0$. Then any radial solution $u = u(r)$, $r = |x|$, of the equation is shown to be classified into one of several types according to its behavior as $r \rightarrow 0$ and $r \rightarrow \infty$. Under the assumption that f is supercritical for small $u > 0$ and is subcritical for large $u > 0$, we clarify the entire structure of the set of solutions of various types. The Pohozaev identity plays a crucial role in the investigation of the structure.

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Burgers-Korteweg-de Vries-type equation and its first integrals

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In this talk, we study a Burgers-Korteweg-de Vries-type equation. The Poincaré phase plane analysis

reveals that this equation has neither nontrivial bell-profile traveling solitary waves, nor periodic waves. We will present some recent results on the first integral by the Lie symmetry method. Our study indicates that the Burgers-Korteweg-de Vries-type equation indirectly admits one-parameter Lie groups of transformations with certain parametric conditions and a class of solitary wave solutions with arbitrary velocity is obtained accordingly.

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Positive solutions to a semilinear elliptic equation with a singular nonlinearity

Galina Grishina

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We consider a second order elliptic equation with measurable bounded coefficients

$$(a_{ij}(x)u_{x_i})_{x_j} + p(x)|x|^s u^{-\sigma} = 0 \quad x \in \Omega \setminus \{O\},$$

where $\sigma > 0$, s is any real number, and $\Omega \subset \mathbf{R}^n$ is a bounded domain, which contains the origin O .

The existence, nonexistence and behaviour of positive weak solutions near the point O will be discussed.

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Microbial competition for a single nutrient with internal storage in an unstirred chemostat

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Jifa Jiang and Feng-Bin Wang

In this talk we present a mathematical model of two microbial species competing for a single-limited nutrient with internal storage in an unstirred chemostat. First we establish the global stability of a steady state for a single population. Then the asymptotic behavior of solutions of the reaction-diffusion system of PDEs for two competing species is established as function of parameters. It is determined when neither, one, or both competing populations

survive. Techniques include maximum principle, upper and lower solution method and the theory of monotone dynamical systems.

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Multiple stable patterns in a balanced bistable equation with heterogeneous environments

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Hiroshi Matsuzawa

In this talk, we report recent studies on the existence of stable solutions with inner transition layers to the problem:

$$\epsilon^2 \Delta u + h(x)^2 (a(x)^2 - u^2) u = 0, \quad x \in \Omega,$$

$$\frac{\partial u}{\partial n} = 0, \quad x \in \partial \Omega$$

for sufficiently small $\epsilon > 0$ under certain assumptions on $h(x)$ and $a(x)$, where Ω is a smooth bounded domain in R^N .

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Global result for multiple positive radial solutions of p-Laplacian system on exterior domain

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Yong-Hoon Lee

In this talk, we consider the existence, nonexistence and multiplicity of positive solutions for p-Laplacian system which has singular indefinite weights. We prove the three solutions theorem for the singular system and use this theorem to show the multiplicity.

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Singular Solutions of the Vlasov-Poisson System

Yi Li

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Professor Jurgen Batt, University of Munich, Germany

In this talk we study the positive solutions of the

generalized Matukuma partial differential equation. T. Matukuma proposed a special case in 1935 for the description of certain stellar globular clusters in a steady state. They correspond to time-independent solutions of the Vlasov-Poisson system in the case of spherical symmetry.

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On the multi-peak stationary solutions to the Gierer-Meinhardt system with weak saturation and source term

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We are concerned with stationary solutions to the following Gierer-Meinhardt system with saturation and source term:

$$\begin{cases} A_t = \epsilon^2 \Delta A - A + \frac{A^2}{H(1+kA^2)} + \sigma_0 & \text{in } \Omega \times (0, \infty), \\ \tau H_t = D \Delta H - H + A^2 & \text{in } \Omega \times (0, \infty), \\ \frac{\partial A}{\partial \nu} = \frac{\partial H}{\partial \nu} = 0 & \text{on } \partial \Omega \times (0, \infty). \end{cases} \quad (1)$$

We suppose Ω is an x_N -axially symmetric domain and $\sigma_0 \geq 0$ is an x_N -axially symmetric function of class $C^\alpha(\bar{\Omega})$. For sufficiently small ϵ and sufficiently large D , we construct a multi-peak stationary solution under the condition that $k\epsilon^{-2N}$ converges to some $k_0 \in [0, \infty)$ as $\epsilon \rightarrow 0$.

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The global dynamics of p-Laplacian with absorption

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Xinfu, Chen and Mingxin Wang

In this talk, I shall report some recent results on how to use self-similarity transformation and a priori estimate to derive sharp asymptotic dynamics of parabolic p-Laplacian with absorption.

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Ideal and Defective Solutions to a Free Boundary Problem from Block Copolymer Morphology

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Xiaosong Kang and Juncheng Wei

The Ohta-Kawasaki density functional theory of diblock copolymers gives rise to a nonlocal free boundary problem. In a proper parameter range an equilibrium pattern of many droplets is proved to exist in a general planar domain. A sub-range is identified where the multiple droplet pattern is stable. Each droplet is close to a round disc. The boundaries of the droplets satisfy an equation that involves the curvature of the boundary and a quantity that depends nonlocally on the whole pattern. The locations of the droplets are determined via a Green's function of the domain. A more involved analysis also shows the existence of a ring pattern solution on a general domain.

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Positive solutions for a class of p-Laplacian systems with multiple parameters

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R. Shivaji

Abstract

Consider the system

$$-\Delta_p u = \lambda_1 f(v) + \mu_1 h(u) \text{ in } \Omega$$

$$-\Delta_q v = \lambda_2 g(u) + \mu_2 \gamma(v) \text{ in } \Omega$$

$$u = 0 = v \text{ on } \partial\Omega$$

where $\Delta_s z = \operatorname{div}(|\nabla z|^{s-2} \nabla z)$, $s > 1$, $\lambda_1, \lambda_2, \mu_1$ and μ_2 are non-negative parameters, and Ω is a bounded domain in R^n with smooth boundary $\partial\Omega$. We prove the existence of a large positive solution for $\lambda_1 + \mu_1$ and $\lambda_2 + \mu_2$ large when

$$\lim_{x \rightarrow \infty} \frac{f(M[g(x)]^{1/q-1})}{x^{p-1}} = 0$$

for every $M > 0$, $\lim_{x \rightarrow \infty} \frac{h(x)}{x^{p-1}} = 0$ and

$\lim_{x \rightarrow \infty} \frac{\gamma(x)}{x^{q-1}} = 0$. In particular, we do not assume any sign conditions on $f(0), g(0), h(0)$ or $\gamma(0)$. We also discuss a multiplicity results when $f(0) = g(0) = h(0) = \gamma(0) = 0$.

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n x n Laplacian Systems with Combined Non-linear Effects

Ratnasingham Shivaji

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Jaffar Ali and K.J. Brown

We study the existence and multiplicity of positive solutions to $n \times n$ systems of the form

$$-\Delta u_1 = \lambda f_1(u_2) \text{ in } \Omega$$

$$-\Delta u_2 = \lambda f_2(u_3) \text{ in } \Omega$$

$$\vdots = \vdots$$

$$-\Delta u_{n-1} = \lambda f_{n-1}(u_n) \text{ in } \Omega$$

$$-\Delta u_n = \lambda f_n(u_1) \text{ in } \Omega$$

$$u_1 = u_2 = \dots = u_n = 0 \text{ on } \partial\Omega.$$

Here Δ is the Laplacian operator, λ is a positive parameter, Ω is a bounded domain in R^n with smooth boundary $\partial\Omega$ and $f_i \in C^1([0, \infty))$, $i \in \{1, 2, \dots, n\}$ belongs to a class of strictly increasing functions that have a combined sublinear effect at ∞ . We establish our results by the method of sub and super solutions.

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Mathematical model for maintenance of germline stem cell niche

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Ting Xie

In the ovarian niche of drosophila, there are two to three germline stem cells. They compete each other for occupancy. The experiment data show cadherin serve as a signal molecule to play an important role in this process. We use mathematical model to study the competition within the niche and the maintenance of the niche.

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Asymptotic Symmetry of nonlinearly stable solutions for parabolic equations

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This talk is mainly concerned with asymptotic symmetry of nonlinearly stable solutions for parabolic

equations by approach of monotone dynamical systems under a group action. These results are then applied to study the spatio-temporal asymptotics of stable solutions of reaction-diffusion equations on a symmetric domain in both time independent and recurrent structures. In particular, dynamical behavior is established for nonlinearly stable bounded solutions of R-D equations on a ball.

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Cross-diffusion induced instability and stability in reaction-diffusion system

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Junping Shi

In a reaction-diffusion system, diffusion can induce the instability of a uniform equilibrium which is stable with respect to a constant perturbation, as shown by Turing in 1950s. We show that cross-diffusion can destabilize a uniform equilibrium which is stable for the kinetic and self-diffusion reaction systems; on the other hand, cross-diffusion can also stabilize a uniform equilibrium which is stable for the kinetic system but unstable for the self-diffusion reaction system. Application is given to vegetation pattern formation in a water-limited ecosystem.

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Minimization of the principal eigenvalue and its applications to population dynamics

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Chiu-Yen Cao and Yuan Lou

This talk is concerned with an indefinite weight linear eigenvalue problem in cylindrical domains. We investigate the minimization of the positive principal eigenvalue under the constraint that the weight is bounded by a positive and a negative constant and the total weight is a fixed negative constant. Biologically, this minimization problem is motivated by the question of determining the optimal spatial arrangement of favorable and unfavorable regions for a species to survive. Both our analysis and numerical simulations for rectangular domains indicate that there exists a threshold value such that if the total weight is below this threshold value then the optimal favorable region is a circular type domain at one of the four corners, and a strip at the one end with shorter edge otherwise.

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Positive Solutions For A Class Of Infinity Semipositone Problems

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Mythily Ramaswamy and R. Shivaji

We analyze the positive solutions to the singular boundary value problem

$$-\Delta u = \lambda[f(u) - 1/u^\alpha]; x \in \Omega$$

$$u = 0; x \in \partial\Omega,$$

where f is a C^2 function in $(0, \infty)$, $f(0) \geq 0$, $f' > 0$, $\lim_{s \rightarrow \infty} \frac{f(s)}{s} = 0$, λ is a positive parameter, $\alpha \in (0, 1)$ and Ω is a bounded region in R^n , $n \geq 1$ with $C^{2+\gamma}$ boundary for some $\gamma \in (0, 1)$. In the case $n = 1$ we use the quadrature method and for $n > 1$ we use the method of sub-super solution to establish our results.

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