

## Special Session 3: Qualitative Behavior of Solutions to Evolutionary PDE's.

Irena Lasiecka, University of Virginia, USA  
Grozdna Todorova, University of Tennessee, Knoxville, USA  
Mitsuhiro Nakao, Kyushu University, Japan

### On the convergence rate of Glimm scheme for general nonlinear hyperbolic systems

**Fabio Ancona**

University of Bologna, Italy  
ancona@ciram.unibo.it

**Andrea Marson**

Consider the Cauchy problem for an  $N$ -dimensional, strictly hyperbolic, quasilinear system

$$u_t + A(u)u_x = 0, \quad u(0, x) = \bar{u}(x)$$

where  $u \mapsto A(u)$  is a smooth matrix-valued map, and the initial data  $\bar{u}$  is assumed to have small total variation. We investigate the rate of convergence of approximate solutions of the above system constructed by the Glimm scheme, under the assumption that, for each  $k$ -th characteristic family, the linearly degenerate manifold

$$\mathcal{M}_k \doteq \{u \in \Omega : D\lambda_k(u) \cdots_k(u) = 0\}$$

( $\lambda_k(u)$ ,  $r_k(u)$  being the  $k$ -th eigenvalue and a corresponding eigenvector of  $A(u)$ ) is either the whole space or consists of a finite number of smooth,  $N - 1$ -dimensional, connected, manifolds that are transversal to the characteristic vector field  $r_k$ . Relying on an adapted wave tracing method, we obtain the same type of error estimates valid for hyperbolic systems satisfying the classical assumptions of genuine nonlinearity or linear degeneracy of the characteristic families.

→ ∞ ◊ ∞ ←

### Global well-posedness for the Maxwell-Schroedinger system

**Ioan Bejenaru**

Texas A&M University, USA  
bejenaru@math.tamu.edu

**D. Tataru**

We are dealing with the global well-posedness for the Maxwell Schroedinger system with the initial data in the energy class. We establish global well-posedness.

→ ∞ ◊ ∞ ←

### Generalized gradient-like dynamical systems under perturbation

**Alexandre Carvalho**

Universidade de São Paulo, Brazil  
andcarva@icmc.usp.br

**J. A. Langa**

We introduce the concept of *gradient-like semigroup* as an intermediate concept between a *gradient* semigroup (possessing a *Liapunov function*) and a semigroup possessing a *gradient-like attractor*. We prove that a perturbation of a gradient-like semigroup is remains a gradient-like semigroup, introduce the concept of gradient-like evolution process and prove that a non-autonomous perturbation of a gradient-like semigroup is a gradient-like evolution process. For gradient-like semigroups and evolution processes, we prove continuity, characterization and exponential attraction of their attractors under perturbation extending the known results on the subject.

→ ∞ ◊ ∞ ←

### Asymptotic decay rates for a solution of a nonlinear hyperbolic system

**Moez Daoulatli**

University of Sousse and LAMSIN, Tunisia  
moez.daoulatli@infcom.rnu.tn

**I. Lasiecka and D. Toundykov**

In this talk we discuss the asymptotic decay rate of a nonlinear dissipative hyperbolic system in bounded domain. Where the damping is localized and modeled by a continuous monotone function without growth assumptions imposed on the nonlinearity. In particular we present a method that reduce the study of the rate of decay of the energy of the nonlinear problem to an observability inequality verified by an associated linear system and a nonlinear differential equation of monotone type.

→ ∞ ◊ ∞ ←

### Some results on global controllability of Burgers equation

**Oleg Emanouilov**

Colorado State University, USA  
oleg@math.colostate.edu

We present some recent results on the global controllability of the Burgers equation and its generalization for 2-d case.

→ ∞ ◊ ∞ ←

### Fundamental solutions for wave equation in Robertson-Walker models of universe

**Anahit Galstyan**

University of Texas-Pan American, USA  
agalstyan@utpa.edu

**Karen Yagdjian**

We introduce the fundamental solutions for the wave equation arising in the Robertson-Walker models of the universe, in particular, for the wave equation in the de Sitter and anti-de Sitter spacetime. We use these fundamental solutions to write the explicit formulas for the solutions of the initial value problem. The last formulas allows us to prove the  $L_p$  -  $L_q$ -decay estimates for the solutions of the equation with and without a source term in the anti-de Sitter background. This is a joint work with Karen Yagdjian (UTPA).

→ ∞ ◊ ∞ ←

### Weighted Strichartz estimates and well-posedness for nonlinear wave equations with low-regularity data

**Kunio Hidano**

Mie University, Japan  
hidano@edu.mie-u.ac.jp

The first part of this talk is concerned with the global in time Strichartz-type estimates for the free wave equation which are derived in the two different ways:(1) the direct estimate of the explicit formula with the help of the Tomas-Stein lemma and (2) the interpolation between the trace inequality and the Morawetz-type estimate. The second part is concerned with the global or local in time Strichartz-type estimates for radially symmetric solutions which are derived from Morawetz-type estimates via weighted Hardy-Littlewood-Sobolev inequalities. Under radial symmetry we get significant gains over the usual Strichartz estimate. Some applications to nonlinear equations are also addressed.

→ ∞ ◊ ∞ ←

### Local energy decay for a class of hyperbolic equations

**Ryo Ikehata**

Hiroshima University, Japan  
ikehatar@hiroshima-u.ac.jp

**Shintaro Aikawa**

We will introduce our recent results concerning local energy decay for a class of hyperbolic equations in an exterior domain. The two methods due to Ikehata-Matsuyama (1999) and Todorova-Yordanov (2001) are effectively applied to our results. Our results will be a partial extension of that derived by Zachmanoglou (1966) to the hyperbolic equations.

→ ∞ ◊ ∞ ←

### Some well-posedness results in nonlinear acoustics

**Barbara Kaltenbacher**

University of Stuttgart, Germany  
barbara.kaltenbacher@mathematik.uni-stuttgart.de

**Irena Lasiecka and Slobodan Veljović**

Motivated by a medical application from lithotripsy, we study the Westervelt equation

$$-\frac{1}{c^2}D_t^2u + \Delta u + \frac{b}{c^2}\Delta(D_tu) = -\frac{\beta_a}{\rho c^4}D_t^2u^2$$

in  $(0, T) \times \Omega$

$$\frac{\partial u}{\partial n} = g \quad \text{on } (0, T) \times \Gamma_0$$

$$\frac{\partial u}{\partial n} = 0 \quad \text{on } (0, T) \times (\partial\Omega \setminus \Gamma_0)$$

$$u(0, \cdot) = D_tu(0, \cdot) = 0 \quad \text{in } \Omega$$

in a bounded domain  $\Omega$ . This models the nonlinear evolution of the acoustic pressure  $u$  excited at a part  $\Gamma_0$  of the boundary. Whereas shock waves are known to emerge after a sufficiently large time interval for appropriate initial and boundary conditions, we here prove existence and uniqueness as well as stability of a solution  $u$  for small data  $g$  or short time  $T$ , using a fixed point argument. Moreover we discuss extension of the result to absorbing boundary conditions as well as to the more general model given by the Kuznetsov equation

$$D_t^2\psi - c^2\Delta\psi = D_t \left( b\Delta\psi + \frac{1}{c^2} \frac{B}{2A} (D_t\psi)^2 + |\nabla\psi|^2 \right)$$

for the acoustic velocity potential  $\psi$ . Numerical simulations will illustrate the theoretical results.

→ ∞ ◊ ∞ ←

## Decay estimates for dissipative wave equations with space-time dependent potential

**Jonathan Kenigson**

University of Tennessee, USA  
jkenigso@utk.edu

**Jessica Kenigson**

We study the long time behavior of solutions of dissipative wave equations with space-time dependent potential. When the potential is only time-dependent, Fourier analysis is a powerful tool to derive sharp decay estimates for solutions. When the potential is only space-dependent, a powerful new technique has been developed by Todorova and Yordanov to capture the exact decay of solutions. The presence of a space-time dependent potential, as in our case, requires modification of this technique. We found the energy decay and decay of the  $L^2$  norm of solutions in the case of space-time dependent potential. These estimates help investigate the existence and behavior of global solutions for various nonlinear perturbations.

→ ∞ ◊ ∞ ←

## Nonlinear Schrödinger equations with complex nonlinear coefficient

**Naoyasu Kita**

University of Miyazaki, Japan  
nkita@cc.miyazaki-u.ac.jp

This talk partly consists of the joint work with A. Shimomura in Tokyo Metropolitan University. In this talk, the initial value problem for nonlinear Schrödinger equations will be discussed, where the space dimension is one and the nonlinearity is of well-known gauge invariant power type. When the nonlinear coefficient is purely real number, many authors have so far studied the asymptotic behavior of the solution. But, in this talk, we consider the case where the nonlinear coefficient contains imaginary part. This model is often said to express the dynamics of electro-magnetic waves propagating through nonlinear optical media where the energy loss caused by electric current is in consideration. We first observe large-time behavior of the solution for small or large initial data when the nonlinear coefficient contains negative imaginary part. As a byproduct of this asymptotic result, the existence of a blowing-up solution will be also presented.

→ ∞ ◊ ∞ ←

## On the boundary stabilization of some hyper-

## bolic systems

**Vilmos Komornik**

Université de Strasbourg, France  
vilmos.komornik@gmail.com

We compare two constructive methods to stabilize hyperbolic systems by suitably chosen boundary feedbacks. Both are based on the multiplier method. The first one leads to simpler feedbacks but its applicability is narrower and the decay rates are limited. The second one provides more sophisticated feedbacks in more general situations and arbitrarily high decay rates can be achieved.

→ ∞ ◊ ∞ ←

## An elementary proof of Global existence for nonlinear wave equations in an exterior domain

**Hideo Kubo**

Osaka University, Japan  
kubo@math.sci.osaka-u.ac.jp

In this talk, we consider the exterior problem for the nonlinear wave equations. The main aim of this talk is to present an alternative proof of global existence result due to Metcalfe, Nakamura and Sogge (Japan. J. Math. (N.S.), 2005). Our approach is based on the weighted pointwise estimates transferred from the corresponding estimates for the Cauchy problem and on the enhanced decay estimate for the tangential derivative to the light cone. The former enables us to exploit the stronger decay property than the preceding works, while, the latter is adopted to extract additional decaying factor from the null form. In addition, asymptotic behavior of solutions for the exterior problem will be discussed.

→ ∞ ◊ ∞ ←

## Quasilinear parabolic systems with fully nonlinear boundary conditions

**Yuri Latushkin**

University of Missouri-Columbia, USA  
yuri@math.missouri.edu

We study quasilinear systems of parabolic partial differential equations with fully nonlinear boundary conditions on bounded or exterior domains. Our main results concern the asymptotic behavior of the solutions in the vicinity of an equilibrium. The local center, center-stable, and center-unstable manifolds

are constructed and their dynamical properties are established using nonautonomous cutoff functions. Under natural conditions, we show that each solution starting close to the center manifold converges to a solution on the center manifold. This is a joint project with Jan Pruss and Roland Schnaubelt.

→ ∞ ◊ ∞ ←

## Hopf-Lax type formulas and related problems

**Paola Loreti**

Sapienza University of Rome, Italy

loreti@dmmm.uniroma1.it

**Antonio Avantaggiati**

In this talk we shall deal with Hopf-Lax type formulas, viscosity solutions to evolutive, first order Hamilton-Jacobi equations, hypercontractivity properties, then we shall briefly deal with logarithmic Sobolev inequalities.

→ ∞ ◊ ∞ ←

## Energy decay to the Cauchy problem of nonlinear Klein-Gordon equations with a sublinear dissipative term

**Mitsuhiro Nakao**

Kyushu University, Japan

mnakao@math.kyushu-u.ac.jp

We derive decay estimates of energy for solutions of nonlinear Klein-Gordon equation  $u_{tt} - \Delta u + u + \rho(u_t) + g(u) = 0$  in  $R^N \times (0, \infty)$ . Here the dissipative term  $\rho(u_t)$  behaves like  $\rho(u_t) \approx |u_t|^r u_t, -1$ .

→ ∞ ◊ ∞ ←

## Global solutions to the Cauchy problem for the system of damped wave equations

**Takashi Narazaki**

Tokai University, Japan

narazaki@ss.u-tokai.ac.jp

In this talk we study the Cauchy problem for the weakly coupled system of the following damped wave equations  $u_{tt} - \Delta u + u_t = |v|^{\sigma_1} v, v_{tt} - \Delta v + v_t = |u|^{\sigma_2} u, t > 0, x \in R^n$ , where  $\sigma_1, \sigma_2 \in [1, \infty)$ . Under the assumption that  $\max((\sigma_1 + 1)/(\sigma_1 \sigma_2 - 1), (\sigma_2 + 1)/(\sigma_1 \sigma_2 - 1)) < n/2p, \sigma_1 \sigma_2 > 1, p \in [1, \infty), n = 1, 2, 3$ , we show the global existence of solutions to the above problem, provided that and initial data  $(u(0), u_t(0)) \in (W^{1,p} \cap W^{1,\infty}) \times L^p \cap L^\infty,$

$(v(0), v_t(0)) \in (W^{1,p} \cap W^{1,\infty}) \times L^p \cap L^\infty$  are sufficiently small. Moreover, we will also show the asymptotic behavior of the solutions. Here  $L^p := L^p(R^n)$  and  $W^{m,p} := W^{m,p}(R^n)$ .

→ ∞ ◊ ∞ ←

## Behavior of solutions for the semilinear heat equation and damped wave equation with slowly decaying data

**Kenji Nishihara**

Waseda University, Japan

kenji@waseda.jp

We consider the Cauchy problem for the semilinear heat equation

$$\begin{cases} \phi_t - \Delta \phi = |\phi|^{\rho-1} \phi, & (t, x) \in \mathbf{R}_+ \times \mathbf{R}^N \\ \phi(0, x) = \phi_0(x), & x \in \mathbf{R}^N, \end{cases}$$

and for the damped wave equation

$$\begin{cases} u_{tt} - \Delta u + u_t = |u|^{\rho-1} u, & (t, x) \in \mathbf{R}_+ \times \mathbf{R}^N \\ (u, u_t)(0, x) = (u_0, u_1)(x), & x \in \mathbf{R}^N, \end{cases}$$

with  $\rho > 1$ . Here, the initial data are especially assumed to be

$$\phi_0, u_0, u_1 \sim (1 + |x|^2)^{-k/2}, \quad 0$$

→ ∞ ◊ ∞ ←

## Remarks on proof of virial identity for nonlinear Schrödinger equations

**Masahito Ohta**

Saitama University, Japan

mohta@mail.saitama-u.ac.jp

The finite time blowup of solutions for nonlinear Schrödinger equations is based on the virial identity. We give a simple proof of the virial identity in an abstract setting.

→ ∞ ◊ ∞ ←

## Attractors for semilinear equations of viscoelasticity

**Vittorino Pata**

Politecnico di Milano, Italy

vittorino.pata@polimi.it

**S. Gatti, A. Miranville and S. Zelik**

We analyze a differential system arising in the theory of isothermal viscoelasticity, equivalent to the integro-differential equation of hyperbolic type

$$\partial_{tt}u - \Delta u + \int_0^\infty \mu(s)\Delta u(t-s)ds + \phi(u) = 0,$$

where  $\phi$  is a suitable nonlinearity, and  $\mu$  (called *memory kernel*) is a nonincreasing summable function on  $(0, \infty)$ . The dissipation mechanism is contained only in the convolution integral, accounting for the past history of the displacement. In particular, we consider memory kernels which entail an extremely weak dissipation. In spite of that, we show that the related dynamical systems possess global attractors of optimal regularity.

→ ∞ ◊ ∞ ←

### Energy decay rates of magnetoelastic waves in a bounded (or unbounded) conductive medium

**Gustavo Perla menzala**

National Laboratory of Scientific Computation, Brazil  
perla@lncc.br

We consider a coupled system of equations arising in the theory of magnetoelasticity in the presence of a nonlinear localized damping (in the bounded domain case). We prove uniform rates of decay of the total energy using multipliers techniques, Nakao's lemma and unique continuation. The exterior domain case is treated only with linear localized damping and additional conditions on the initial data for the elastic part of the system. We adapt recent techniques due to M. Nakao and R. Ikehata for the scalar wave equation to our case.

→ ∞ ◊ ∞ ←

### Uniqueness results in the Cauchy problem for degenerate Kolmogorov equation

**Sergio Polidoro**

Università di Modena e Reggio Emilia, Italy  
sergio.polidoro@unimore.it

**C. Cinti**

We consider the Cauchy problem for hypoelliptic Kolmogorov equations in the form  $\partial_t u = \sum_{i,j=1}^m a_{i,j}(x,t) \partial_{x_i x_j} u + \sum_{j=1}^m a_j(z) \partial_{x_j} u + \sum_{i,j=1}^N b_{i,j} x_i \partial_{x_j} u$  where  $(x,t) \in \mathbb{R}^N \times ]0, T[$ ,  $1 \leq m \leq N$ , as well as in its divergence form. We prove that,

if  $|u(x,t)| \leq M \exp(a(t^{-\beta} + |x|^2))$ , for some positive constants  $a, M$  and  $\beta \in ]0, 1[$  and  $u(\cdot, 0) \equiv 0$ , then  $u \equiv 0$ . The proof of the main result is based on some previous uniqueness result and on the application of some "estimates in short cylinders", first introduced by Safonov in the study of uniformly parabolic operators.

→ ∞ ◊ ∞ ←

### Decay rates for damped wave equations with variable coefficients

**Petronela Radu**

University of Nebraska-Lincoln, USA  
pradu@math.unl.edu

**Grozdena Todorova and Boris Yordanov**

The authors establish weighted decay rates for the energy associated with the Cauchy problem

$$\begin{cases} u_{tt} - \operatorname{div}(b(x)\nabla u) + a(x)u_t = 0, & x \in \mathbb{R}^n, t > 0 \\ u(0, x) = u_0(x), \quad u_t(0, x) = u_1(x), & x \in \mathbb{R}^n. \end{cases}$$

Such a system appears in models for traveling waves in a non-homogeneous gas with damping that changes with the position. It is also a model for hyperbolic heat conduction when the heat flux changes with the position. This problem has been studied intensively for the homogeneous medium (when  $b$  is constant), but the results are scarce for the variable coefficient case. In fact, to the authors' knowledge, the results of this paper are the first to be obtained for wave equations with variable coefficients on the entire space  $\mathbb{R}^n$  when damping is present. The proof is based on the multiplier method where the multiplier is cleverly engineered from an asymptotic profile of solutions. In addition, we will present decay rates for higher energies associated with the system. These results were obtained in collaboration with G. Todorova and B. Yordanov from University of Tennessee, Knoxville.

→ ∞ ◊ ∞ ←

### The influence of damping and source terms on solutions of nonlinear wave equations

**Mohammad Rammaha**

University of Nebraska-Lincoln, USA  
mrammaha1@math.unl.edu

In this talk we present some recent results on the existence, uniqueness and blow up in finite time of solutions to nonlinear wave equations. In the case of a single wave equation or systems of wave equations,

there two competing forces present. One force is a degenerate damping term and the other is a strong source. In particular, we analyze influence of these forces on the long-time behavior of solutions.

→ ∞ ◊ ∞ ←

### Buckling instabilities and global continuation in nonlinear elasticity

**Henry Simpson**

University of Tennessee, USA  
hsimpson@math.utk.edu

Bifurcation phenomena in nonlinear elastostatics is investigated. In particular, a rectangular rod is compressed between frictionless plates leading to barrelling or buckling of the rod. Linearization instabilities of the system of nonlinear elliptic partial differential equations governing an elastic material yield pitchfork bifurcation of a branch of solutions from a known trivial (homogeneous) branch. Boundary conditions are of mixed and/or nonlinear type. Rigorous proof is provided via the local implicit function theorem and a degree theory for global branching for proper, Fredholm maps in a Sobolev space setting. We report spectral properties of the linearized operator and their relation to an infinite sequence of bifurcation points along the trivial branch. These accumulate at places where the complementing and Agmon's conditions fail at a boundary point of the rod.

→ ∞ ◊ ∞ ←

### Critical exponent problem for nonlinear dissipative wave equations with space-dependent potential

**Grozdena Todorova**

University of Tennessee, USA  
todorova@math.utk.edu

**Ryo Ikehata and Borislav Yordanov**

We study the balance between the effect of spatial inhomogeneity of the potential in the dissipative term and the focusing nonlinearity. Sharp critical exponent results will be presented in both cases of fast and slow decaying potential.

→ ∞ ◊ ∞ ←

### Finite-dimensionality and smoothness of the global attractor for a semilinear wave equation with localized interior or boundary damping

### and a critical source term

**Daniel Toundykov**

University of Nebraska-Lincoln, USA  
daniel.toundykov@gmail.com

**Igor Chueshov and Irena Lasiecka**

This work addresses existence and properties of the attractor for a semi-linear damped wave equation with a source term. The dissipation is modeled by a nonlinear monotone velocity feedback supported only on a *subset of the interior or a subset the boundary*. The source is represented by the Nemytski operator associated to a nonlinear map whose growth order may include (in dimensions above 2) the corresponding *critical* Sobolev exponent (for the embedding  $H^1 \rightarrow L^2$ ). It is known that existence of attractors is strictly linked with asymptotic compactness of evolution trajectories, while at the level of critical exponents the compactness of Sobolev embeddings is lost. The study of attractors under critical damping and sources is a challenging problem, and a comprehensive treatment of such models appeared only recently in the literature. The results so far, however, have focused on full interior or full boundary damping. In this case we investigate the issue of *critical exponents combined with geometrically restricted dissipation*. Using Carleman-type estimates and abstract results on dynamical systems, we prove that such an evolution may possess a smooth global compact attractor of a finite fractal dimension.

→ ∞ ◊ ∞ ←

### The semilinear Klein-Gordon equation in de Sitter spacetime

**Karen Yagdjian**

University of Texas-Pan American, USA  
yagdjian@utpa.edu

In the talk we introduce the fundamental solutions for the Klein-Gordon equation in the de Sitter spacetime. These fundamental solutions are used to find representations of the solutions to the Cauchy problem for equation with and without source term. For the equation with large mass we obtain the  $L_p$ - $L_q$  estimates. Next we consider the issue of existence of global in time solution of the semilinear Klein-Gordon equation in de Sitter spacetime.

→ ∞ ◊ ∞ ←

### Global smooth solutions of IBVP to nonlinear suspended string equations

**Masaru Yamaguchi**  
Tokai University, Japan  
edel\_shounifty.com

Let  $\Omega = (0, a) \times (0, T)$ . Consider IBVP to a nonlinear equation of a flexible and heavy suspended string of finite length  $a$  with uniform density

$$\partial_t^2 u(x, t) + L u(x, t) + f(x, u(x, t)) = 0, \quad (x, t) \in \Omega,$$

$$u(a, t) = 0, \quad t \in (0, T),$$

$$u(x, 0) = \phi(x), \quad \partial_t u(x, 0) = \psi(x), \quad x \in (0, a),$$

where  $L = L_0(x, \partial_x)$  is a second order differential operator of the form

$$L = -(x \partial_x^2 + \partial_x),$$

$f(x, u)$  is smooth in  $(x, u) \in [0, a] \times R^1$  and of order 3 in  $u$  at  $u = 0$ , and  $\phi, \psi$  belong to suitable function spaces and lie in the potential well. The purpose of this talk is to present that the IBVP has a time global smooth solution and to study the regularity of the solutions. It is shown in joint work with J. Wongsawasdi that IBVP has a time-global weak solution under weaker conditions with less smoothness on  $f$  and  $\phi, \psi$ . and that IBVP with monotone  $f(x, u) = u^3$  has a time-global classical solution under smooth initial values.

→ ∞ ◊ ∞ ←

### Singular limit for Kirchhoff type quasilinear hyperbolic equation with weak dissipation

**Taeko Yamazaki**  
Tokyo University of Science, Japan  
yamazaki\_takoma.noda.tus.ac.jp

Let  $H$  be a separable Hilbert space with norm  $\|\cdot\|$ . Let  $A$  be a non-negative self-adjoint operator with domain  $D(A)$ . We are concerned with a quasilinear hyperbolic equation of Kirchhoff type with dissipation term  $\varepsilon u_\varepsilon''(t) + b(t)u_\varepsilon'(t) + m(\|A^{1/2}u_\varepsilon(t)\|^2)Au_\varepsilon(t) = 0$ ,  $u_\varepsilon(0) = u_{(0)}$ ,  $u_\varepsilon'(0) = u_{(1)}$ , where  $m \in C^1([0, \infty))$  with  $\inf_{r \geq 0} m(r) > 0$ , and  $b(t)$  is a  $C^1$  function on  $[0, \infty)$  satisfying  $b_0(1+t)^{-p} \leq b(t) \leq b_1(1+t)^{-p}$ ,  $|b'(t)| \leq b_2(t+1)^{-p-1}$  with  $0 \leq p < 1$ ,  $b_0, b_1, b_2 > 0$ . Then we consider the singular limit problem of the above hyperbolic equation to the corresponding parabolic equation  $b(t)u'(t) + m(\|A^{1/2}u_\varepsilon(t)\|^2)Au_\varepsilon(t) = 0$ ,  $u(0) = u_{(0)}$ . We show time decay estimates of the difference  $u_{vare}(t) - u(t)$  and  $u'_{vare}(t) - u'(t)$ ,

together with the unique global existence of the solution  $u_{vare}(t)$ .

→ ∞ ◊ ∞ ←

### Stochastic PDEs with discontinuous perturbations

**Jerzy Zabczyk**  
Institute of Mathematics Polish Academy of Sciences, Poland  
zabczyk@impan.gov.pl

We report on recent existence results for evolution equations with Levy type perturbations. Some structural and asymptotic properties of their solutions will be discussed as well. The presentation will be based on joint research with E. Priola and S. Peszat and on some material from the book: S. Peszat and J. Zabczyk "Stochastic Partial Differential Equations with Levy Noise", Cambridge University Press, 2007

→ ∞ ◊ ∞ ←

### Solvability conditions for some semi-linear parabolic equations

**Qi Zhang**  
University of California-Riverside, USA  
qizhang@math.ucr.edu  
**James M. Wrkich**

Consider the Cauchy problem of the semilinear parabolic equation

$$\Delta u - \partial_t u + V u^p + w = 0, \quad p > 1,$$

in  $\mathbb{R}^N \times (0, T)$ ,  $T > 0$ ,  $u(x, 0) = u_0(x) \geq 0$ , and  $u_0, V, w \in L^1_{loc}(\mathbb{R}^N)$ . We establish a necessary and nearly sufficient condition on the nonlinear potential  $V = V(x)$  so that the above has a positive solution for some  $w = w(x) \geq 0$ ,  $u_0 = u_0(x) \geq 0$ . An application to the case where  $V$  is the important inverse square potential is also given.

→ ∞ ◊ ∞ ←

### Asymptotics of solutions to a nonlinear system with damping

**Zhiyong Zhang**  
University of Alberta, Canada  
zzhang@math.ualberta.ca

In this talk we give some results on the global existence and the asymptotic behavior of solutions to the Cauchy problem for coupled initial data converging

to constant states at infinity. The basic idea is to propose a linear convection-diffusion wave as the decay profile so as to approximate the limiting behavior as  $x$  approaches infinity. The resulting nonlinear system then converges to the linear profile under certain smallness condition of the initial data and perturba-

tion of the original system. We thus show that the evolution equations may be viewed as the composition of a linear diffusion wave and of a parabolic system which converges even faster.

→ ∞ ◊ ∞ ←