

Special Session 41: Dynamical Systems and Spectral Theory

David Damanik, Rice University, USA

Cantor spectrum for Schrödinger operators with potentials arising from generalized skew-shifts

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Artur Avila and David Damanik

We consider continuous $SL(2, \mathbb{R})$ -cocycles over a strictly ergodic homeomorphism which fibers over an almost periodic dynamical system (generalized skew-shifts). We prove that any cocycle which is not uniformly hyperbolic can be approximated by one which is conjugate to an $SO(2, \mathbb{R})$ -cocycle. Using this, we show that if a cocycle's homotopy class does not display a certain obstruction to uniform hyperbolicity, then it can be C^0 -perturbed to become uniformly hyperbolic. For cocycles arising from Schrödinger operators, the obstruction vanishes and we conclude that uniform hyperbolicity is dense, which implies that for a generic continuous potential, the spectrum of the corresponding Schrödinger operator is a Cantor set.

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On the density of uniformly hyperbolic $SL(2, \mathbb{R})$ -valued cocycles.

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In this talk the nature of the spectrum of the quasi-periodic Schrödinger operator is considered: we show that in the discrete case a generic bounded measurable Schrödinger cocycle has Cantor spectrum. In particular, we have that, for a fixed rationally independent frequency, the set of measurable bounded potentials for which the corresponding Schrödinger cocycle admits an exponential dichotomy (is uniformly hyperbolic) is open and dense. We also discuss the non density of the set of hyperbolic $SL(2, \mathbb{R})$ -cocycles in the C^1 -category.

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Hyperbolic dynamics and the spectrum of the Fibonacci Hamiltonian

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Properties of invariant hyperbolic sets of two dimensional diffeomorphisms are very well studied. Due to M. Casdagli, trace map of the Fibonacci Hamiltonian is hyperbolic for large values of the coupling constant. We use this result to relate the properties of the hyperbolic sets and the properties of the spectrum of the Fibonacci Hamiltonian.

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KAM Method and Limit Periodic Potential.

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Young-Ran Lee

We consider the application of KAM (Kolmogorov-Arnold-Moser) method for spectral investigation of the Schrödinger operator $H = -\Delta + V(x)$ with a limit-periodic potential $V(x)$ in dimension two. We prove that the spectrum of H contains a semiaxis and there is a family of generalized eigenfunctions at every point of this semiaxis with the following properties. First, the eigenfunctions are close to plane waves $e^{i(\vec{k}, \vec{x})}$ at the high energy region. Second, the isoenergetic curves in the space of momenta \vec{k} corresponding to these eigenfunctions have a form of slightly distorted circles with holes (Cantor type structure).

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Dynamics of the Toda Flow

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Gerald Teschl

The Toda flow induces a dynamical system on all the Jacobi matrices. We discuss which spectral properties can be recovered from an appropriate limit as time goes to infinity for Jacobi matrices close to the free one.

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Absence of absolutely continuous spectrum for one-dimensional continuum models with aperiodic order

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Steffen Klassert and Peter Stollmann

We consider one-dimensional Schroedinger operators associated to Delone dynamical systems with aperiodic order. We show that the absolutely continuous spectrum is empty. This is based on Kotani theory and provides an analogue of the corresponding result of Kotani for aperiodic subshifts over a finite alphabet.

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Kotani's theory for random linear hamiltonian systems

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Carmen Núñez

Kotani's theory for n-dimensional Schrödinger equations with perturbation I_n was developed by Kotani and Simon in 1988. We describe some Kotani's type results for families of linear Hamiltonian systems when an Atkinson's perturbation is considered.

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Strange Non-Chaotic Attractors and the spectrum of the Dirac, Schrödinger and Jacobi operators

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Angel Jorba, Rafael Obaya and Joan Carles Tatjer

During the last years a lot of attention has been paid to the analysis of the so-called strange non-chaotic attractors (SNA). In this work we show their connection with the well known almost automorphic but not almost periodic minimal sets appearing in the dynamical description of the projective flows induced by two-dimensional linear systems of ODEs, including discrete and continuous one-dimensional Schrödinger equations. We also establish conditions on this type of systems ensuring the occurrence of SNAs. These conditions are based on the

non-uniform hyperbolicity and on some properties of the systems corresponding to extreme points of spectral gaps for an associated spectral problem. The work has been made in collaboration with Angel Jorba, Rafael Obaya, and Joan Carles Tatjer.

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Fractal Properties of Discrepancy Sums

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In his book *Continued Fractions*, Texas A&M Professor Doug Hensley noticed that the sequence

$$x_n = \sum_{i=1}^n p \left(\sqrt{2} - \frac{1}{2} \right),$$

where $p(x)$ is the parity of x (+1, -1, or 0), appears to exhibit fractal-like properties. Using simple ergodic-theoretic techniques, we show that for all x quadratic irrational numbers such that all odd-indexed terms in the continued fraction expansion of x are even will exhibit such behavior (including $x = \sqrt{2}$, whose expansion is given by $[1; 2, 2, 2, \dots]$).

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The absolutely continuous spectrum of Jacobi matrices

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I will report on the following result and some of its ramifications: Given a Jacobi matrix with some absolutely continuous spectrum, take limit points under the shift map. Then these limiting Jacobi matrices are reflectionless on the support of the ac part of the spectral measure. (This very severely restricts the structure of Jacobi operators with ac spectrum.)

This is a development of earlier work of Breimesser and Pearson. It may also be thought of as a deterministic kin of Kotani theory. The consequences include an Oracle Theorem for the coefficients and Denisov-Rakhmanov type theorems.

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Quasicrystals in Wonderland

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D. Lenz

This talk is about joint work with D. Lenz that deals with spectral properties of Hamiltonians that can be used to model quasicrystals. An appropriate class of models is associated with dynamical systems whose points are Delone sets. A variation on B. Simons Wonderland Theorem shows that these operators exhibit a purely singular continuous spectral component for a generic point of the dynamical system.

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Eigenvalue statistics in the Anderson model

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A physical conjecture says that eigenvalue statistics can be used as a finite volume criterion for localization or delocalization in Anderson models. We will report on rigorous results which confirm that finite volume eigenvalues satisfy Poisson statistics in localized energy regimes.

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Dynamical bounds for the strongly coupled Fibonacci Hamiltonian via complex analysis methods

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David Damanik

We present the approach to upper and lower bounds in quantum dynamics via complex analysis methods. In the case of the Fibonacci Hamiltonian this leads to an optimal description of the time-averaged spreading rate of the fast part of the wavepacket in the large coupling limit. This provides the first example where the time-averaged spreading rates exceed the upper box-counting dimension of the spectrum.

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On a class of Reflectionless Sturm-Liouville potentials

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Russell Johnson

We introduce the class of Reflectionless Sturm-Liouville potentials and the subclass consisting of Sato-Segal-Wilson potentials. We illustrate a procedure to construct potentials of the above type by means of algebro-geometric approximation.

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