Special Session 44: Nonholonomic Constraints in Mechanics and Optimal Control Theory

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Strict abnormal extremals in optimal control problems with nonholonomic constraints.

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The problem to be solved consists of minimizing a functional under nonholonomic constraints. It is understood as an optimal control problem because the unconstrained velocities are considered the controls. There exist different kinds of extremals, that is, curves candidates to be solution, in optimal control theory: abnormal, normal and strictly abnormal. The key point of the abnormal extremals is their independence on the cost function. Whereas, the strict abnormal are abnormal, but not normal, so they depend on the cost function. For many years, they have been discarded as candidates to be optimal up to the appearance of the papers by Montgomery, Liu and Sussmann in the nineties.

The necessary conditions for optimality given by Pontryagin's Maximum Principle from a presymplectic viewpoint allows to start a constraint algorithm, in the sense of Gotay and Nester, useful for characterizing all the set of extremals. In fact, this geometric process can be applied to any control system and any cost function, as long as differentiability with respect to the controls is assumed.

Here, we focus on particular nonholonomic constraints in order to obtain a clear understanding of the strict abnormality in optimal control theory.

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Nonholonomic mechanics, action principles, and control

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In this talk I will discuss various approaches to obtaining the dynamics of nonholonomic systems and relate them to the least action principle of mechanics, and to control theory. I will also discuss the relationship of nonholonomic mechanics to Hamiltonian dynamics and to dissipative dynamics.

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A presymplectic approach to Hamilton-Jacobi-Bellman equation

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Consider the optimal control problem given by a control equation $\dot{x} = \Gamma(x,u)$ and cost function $L = L(x,u)$. In geometric terms we have a control fiber bundle $\pi : C \longrightarrow B$, a vector field Γ along $\pi : \Gamma = \Gamma^i(x,u)\,\frac{\partial}{\partial x^i}$, and L is a function on C. In this paper we describe this problem using the so-called presymplectic formalism, and obtain the corresponding Hamilton-Jacobi-Bellman theory.

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Preservation of an invariant measure in continuous and discrete nonholonomic systems

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Preservation of an invariant measure (IM) in nonholonomic systems may be important for their complete integrability. This talk considers several known and new classes on such systems which possess IM. Some examples of discrete nonholonomic systems preserving an IM will be also presented.

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A momentum-energy integrator for nonholonomic mechanical systems with symmetry

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In this talk we will propose and discuss a geometric integrator for nonholonomic mechanical systems. It has been designed for discrete Lagrangian systems specified through a discrete Lagrangian $L_d \colon Q \times Q \to \mathbb{R}$, where Q is the configuration manifold, and a nonholonomic distribution $\mathcal{D} \subset TQ$. The method does not require to predefine a discretization of the constraints. It preserves the discrete nonholonomic momentum map in the presence of horizontal symmetries, and the nonholonomic constraints are preserved in average. In some important cases, namely when Q is a Lie group and the discrete and continuous Lagrangian have certain invariance properties, the method is also energy-preserving.

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Reduction of Almost Poisson Brackets for Nonholonomic Systems

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Nonholonomic systems are not Hamiltonian. The dynamics can however be described in terms of a bracket of functions that fails to satisfy the Jacobi identity. One now speaks of an almost Poisson bracket. The bracket encodes the forces of constraint and avoids dealing with Lagrange multipliers.

We study the reduction of nonholonomic systems with symmetry from the almost Poisson perspective. I will describe different scenarios for the reduction and discuss how for some important examples the reduced bracket satisfies the Jacobi identity (sometimes after a time reparametrization).

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Geometric Discretization of Nonholonomic Systems with Symmetries

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The talk describes discretization schemes for nonholonomic mechanical systems with symmetries and controllable shape for integration and optimization purposes. The discrete dynamics is derived through the geometric discretization of variational principles. Properties of the resulting discrete momentum and shape dynamics are examined and compared to their continuous analogs. Empirical comparisons with standard methods in terms of accuracy and efficiency will be presented for a simple car model and the snakeboard.

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Nonlinear Dynamics and Stability of the Skateboard

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Analysis and simulation are performed for a simplified model of a skateboard in the absence of rider control. As against to the previous papers, we assume the steering angles of the wheel axles and tilt of the board are finite (not infinitesimal). We find the complete nonlinear relations between these angles. Using these relations we derive equations of motion for two models of the skateboard. To obtain the equations of motion of the skateboard, we use the Gibbs-Appell method. For the simplest two-degree-of- freedom model of the skateboard the problems of integrability of the obtained equations of motion are investigated. It is shown, that for integrability of this system we need, in addition to the energy integral, another first integral. In general case this additional integral is not exists. However it is possible to find, for some values of parameters, the case, when the equations of motion of the skateboard can be completely solve in terms of quadratures. In this case we give the complete analytical investigation of the qualitative behaviour of the skateboard. We study also the problems of bifurcations of steady motions of the skateboard in the integrable case and construct the Poincare and Smale bifurcation diagrams.

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Geometry of variational nonholonomic Lagrangian systems with symmetries

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In this talk, I will present some recent results about the geometric structure of reduced variational nonholonomic Lagrangian systems. In particular, I will discuss optimal control aspects of nonholonomic systems with symmetries and reduction of subriemannian problems.

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New developments in Geometric Integration of Nonholonomic Systems

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In this talk, we will summarize some new numerical methods for for nonholonomic systems. We will find, in different ways, nonholonomic discrete Euler-Lagrange equations in a setting which permits to deduce geometric integrators for continuous nonholonomic systems (reduced or not).

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Non-Abelian Routh reduction and relative equilibria

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Routh's procedure, in its original form, was concerned with eliminating from a Lagrangian problem the generalized velocities corresponding to so-called ignorable or cyclic coordinates. Such cyclic coordinates indicate that there is an Abelian symmetry group for the Lagrangian and they generate conserved quantities, the momentum. We extend this reduction procedure to Lagrangian systems whose symmetry group is not necessarily Abelian. To do so we constrain the Euler-Lagrange field to a level set of momentum in velocity phase space. We present a new method of analysis based on the use of quasi-velocities. We will also discuss the reconstruction of solutions of the full Euler-Lagrange equations from those of the reduced equations and the characterization of relative equilibria in this set-up.

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Towards better understanding the behaviours of rattlebacks

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Rattlebacks are rigid bodies which, when rolled on a table, exhibit a unintuitive spin bias, due to small asymmetries either of the body shape or of the mass distribution. The nonholonomic model consists of a convex body, quite a bit longer than it is wide, rolling without slipping on a plane. It may be possible to understand better the motion of rattlebacks, by viewing the system as a (singular) perturbation from a nonregular nonholonomic system. I will sketch some preliminary work along these lines.

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Conservation laws in non-holonomic mechanics from non-symmetries

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As is well known, Noether's theorem provides a tight link between symmetries and conservation laws for conservative, holonomic mechanical systems. Adding non-conservative forces to the system, on the other hand, or non-holonomic constraints, will generically break the symmetries of the system. So there comes a time when standard methods of symmetry and reduction fail to work for non-holonomic systems. We shall present a geometrical framework for modelling non-holonomic systems in which the algorithmic search for so-called adjoint symmetries works exactly in the same way as in the holonomic case and thus can produce, in principle, all existing first integrals. The computational aspects of the method will be illustrated by some simple examples. A possible extension of the technique to control systems will be explored.

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Vakonomic mechanics on Lie affgebroids

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The geometry and dynamics on Lie algebroids have been extensively studied during the past years. From the Physics point of view, Lie algebroids can be used to give geometric descriptions of Lagrangian and Hamiltonian Mechanics. In the same direction, the cases of nonholonomic and vakonomic mechanics on Lie algebroids has been analized.

A possible generalization of the concept of a Lie algebroid to affine bundles is the notion of a Lie affgebroid. Lie affgebroid structures may be used to develop a time-dependent version of Lagrange and Hamilton equations on Lie algebroids. In the same setting of Lie affgebroids, we have developed a geometric description of Lagrangian systems subject to nonholonomic affine constraints.

In this talk, we pretend to present a geometric description of vakonomic mechanics on Lie affgebroids. First, we introduce a constraint algorithm for precosymplectic Lie algebroids. Then, we deduce the vakonomic equations using our constraint algorithm and study the particular case when it stops in the first step. In this situation, we define the vakonomic aff-Poisson bracket associated with a regular vakonomic system on a Lie affgebroid which plays a prominent role in the description of the vakonomic dynamics.

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A Generalization of the Poincaré-Cartan Integral Invariant for a Nonlinear Nonholonomic Dynamical System

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Based on the d'Alembert-Lagrange-Poincaré variational principle, we formulate general equations of motion for mechanical systems subject to nonlinear nonholonomic constraints, that do not involve Lagrangian undetermined multipliers. We write these equations in a canonical form called the Poincaré-Hamilton equations, and study a version of corresponding Poincaré-Cartan integral invariant which are derived by means of a type of asynchronous variation of the Poincarévariables of the problem that involve the variation of the time. As a consequence, it is shown that the invariance of a certain line integral under the motion of a mechanical system of the type considered characterizes the Poincaré-Hamilton equations as underlying equations of motion. As a special case, an invariant analogous to Poincaré linear integral invariant is obtained.

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Symmetries for nonholonomic field theories

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For nonholonomic systems, the classical theorem of Noether linking symmetries and conservation laws no longer holds true. Instead different authors showed that there exists a so-called non-holonomic momentum equation, describing the evolution of the "conserved" quantities under the nonholonomic flow. In this talk, we extend this work to the case of field theories with nonholonomic constraints, and we derive some previously unknown conserved quantities for field theories and mechanical systems alike.

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Stability of Relative Equilibria of Discrete Nonholonomic Systems

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Cameron Lynch

Center manifold reduction is used to establish asymptotic stability of relative equilibria of discrete nonholonomic systems with symmetry. For systems obtained by discretization of continuous-time dynamics, stability conditions are compared to those of the associated continuous-time systems. The theory is illustrated with the stability analysis of the discrete roller racer. This is a joint work with Cameron Lynch.

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