Special Session 45: Water Waves

Adrian Constantin, Trinity College, Ireland

Symmetry of steady rotational gravity water waves

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Adrian Constantin and Erik Wahlen

We prove that steady periodic two-dimensional rotational gravity water waves with a monotone surface profile between troughs and crests have to be symmetric around the crest, irrespective of the vorticity distribution within the fluid. In addition, we show that deep water waves allow only for vorticity distributions vanishing at large depths. The main result is joint with A. Constantin and E. Wahlen, Lund.

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Shallow water waves with singularities

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We discuss the formation of singularities of several shallow water equations, including the Camassa-Holm and the Degasperis-Procesi equation. We particularly show that the first blow-up of strong solutions must occur in the form of a wave breaking and shocks may possibly appear afterwards.

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The trajectories of particles in deep-water Stokes waves

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We prove that in a nontrivial deep-water Stokes wave there are no closed particle paths. A description of the zero level set of a harmonic function on a semi-infinite domain plays an important role in the proof. A qualitative description of the particle trajectories throughout the fluid is obtained."

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Geometrical Modeling of Dynamics of Non-Linear Flows in Porous Media

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The primary goal of this work is modeling of non-linear dynamic flows in porous media using methods and results of differential geometry. In this work we propose a generalized Darcy equation to model all three Forchheimer equations, the so called two terms law, power law, and three term cubic law. Under some assumptions about the fluid, the generalized Darcy-Forchheimer equations allow reduction of the governing dynamic system of equations to one parabolic non-linear equation for pressure function. This parabolic equation displays some similarity to the constant mean curvature equation for surfaces. In the paper we utilize this similarity to find a constraint on the flows, such that the pressure function can be regarded as a surface with given mean curvature, after a certain stretching and domain transformation. Conversely, a surface with given mean curvature can be interpreted as a pressure distribution of the flow subjected to the Forchheimer equation. This trade off between hydrodynamic and geometrical problems is interesting in itself and can also be used as a homogenization method. The geometric interpretation of the problem of the dynamic flow in porous media provides a new algorithm for the computation of some important hydrodynamic parameters.

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Integrable models for shallow water waves

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The motion of inviscid fluid is described by Euler's equations. In the case of shallow water, one can consider a *perturbative* asymptotic expansion of Euler's equations to a certain order of smallness of the scale parameters. The so obtained asymptotic equations can be matched to certain integrable equations. The best known example in this regard is the KdV

equation.

The aim of the talk is to present some recent results concerning the use of integrable equations, like the *Camassa* C Holm and *Degasperis* C *Procesi* equations, in modeling the motion of shallow water waves.

These equations describe in a direct way the velocity and consequently all related variables of shallow water waves at different depths in the absence of a shear flow. From a modeling point of view, the advantage of the CH and DP equations over *KdV* equation consists in the fact that they capture the wave-breaking phenomenon.

These equations can also be used as water-wave models in the presence of an arbitrary shear flow.

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On free-surface profiles for bounded steady water

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We consider the classical nonlinear problem of steady two-dimensional waves on water of finite depth. In the framework of problem's statement that describes all bounded waves (not necessary periodic or solitary), the following two groups of results will be discussed. The first group deals with the behavior of the free–surface profiles. We proved that: (i) there are no waves when the flow is critical; (ii) there are no waves that have their free-surface profiles totally above or under the upper boundary of the uniform subcritical stream; (iii) two types of behavior of the free surface at infinity (either positive/negative or both) are described. The surface either oscillates infinitely many times around the upper boundary of the subcritical uniform stream or asymptotes the upper level of a uniform stream (subcritical or supercritical).

Second group of results deals with the following two uniqueness statements. Both of them concern sufficiently small waves. First, we demonstrate that for any value of problem's parameter there exists at most one wave such that its free-surface profile asymptotes the upper level of the supercritical uniform stream at positive/negative infinity with a given coefficient in the principal term of asymptotics. Second, we show that for any value of problem's parameter a wave profile that is sufficiently close to the upper level of the subcritical uniform stream is uniquely determined by the values of its height and slope at a certain point which can be chosen arbitrarily. From the obtained results it follows that every sufficiently small steady wave is either a solitary wave or a Stokes wave. All these results are obtained together with Nikolay Kuznetsov.

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On the existence of extreme waves and the Stokes conjecture with vorticity

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We present some recent results on singular solutions of the problem of travelling gravity water waves on flows with vorticity. We show, under general assumptions on the vorticity function, that a wave with stagnation points at its crests can be obtained as a limit of a sequence of regular waves in the continuum whose existence was proved by Constantin and Strauss, provided that certain natural a priori bounds hold along the continuum. The validity of these a priori bounds is discussed. We also show that at a stagnation point about which it is supposed symmetric, the wave profile must have a corner of $120°$ if the vorticity is everywhere nonnegative, and either a corner of $120°$ or a horizontal tangent if the vorticity is everywhere nonpositive.

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Solitary water waves with constant vorticity

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Mark Groves

I will present a new Hamiltonian formulation for water waves with constant vorticity, which is similar to Zakharov's classical formulation in the irrotational setting. I will then show how this formulation can be used to construct solitary capillary-gravity waves using a variational approach known as the reduced energy-momentum method. The method, which was previously used by Buffoni in the context of irrotational flow, gives a form of stability as a side effect. The presence of vorticity requires some non-trivial extensions of Buffoni's proof. The second part of the talk is joint work with Mark Groves.

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Wave-breaking and global existence for the Degasperis-Procesi equation

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In this talk we will discuss several aspects of the formation of singularities and the existence and uniqueness of global solutions for the Degasperis-Procesi equation on the line. We first show that the first blow-up of strong solutions to the equation must occur only in the form of wave-breaking and shock waves possibly appear afterwards. We then present several new blow-up results of strong solutions with certain initial profiles. Moreover, we obtain the global existence of strong solutions to the equation for a class of initial profiles. Furthermore, we prove the existence and uniqueness of global "strong" weak solutions to the equation with certain initial profiles.

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