Special Session 46: Localized Behavior of Elliptic Equations and Systems

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On Schrödinger equations with multisingular inverse-square anisotropic potentials

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In nonrelativistic molecular physics, the interaction between an electric charge and the dipole moment $\mathbf{D} \in R^N$ of a molecule is described by an inverse square potential with an anisotropic coupling strength. In particular the Schrödinger equation for the wave function of an electron interacting with a polar molecule (supposed to be point-like) can be written as

$$\left(-\frac{\hbar^2}{2m}\Delta + e\,\frac{x\cdot\mathbf{D}}{|x|^3} - E\right)\Psi = 0,$$

where e and m denote respectively the charge and the mass of the electron and $\mathbf{D}$ is the dipole moment of the molecule.

We present some results in collaboration with E. Marchini and S. Terracini which describe the asymptotic behavior near the singularity of solutions to equations associated to dipole-type Schrödinger operators of the form

$$-D - \frac{\lambda\,(x\cdot\mathbf{d})}{|x|^3}$$

in R^N, where $N \geq 3$, $\lambda = \frac{2me|\mathbf{D}|}{\hbar}$, being $|\mathbf{D}|$ the magnitude of the dipole moment $\mathbf{D}$, and $\mathbf{d} = \mathbf{D}/|\mathbf{D}|$ denotes the orientation of $\mathbf{D}$. We discuss some applications of the estimate of such a behavior to the analysis of fundamental properties of Schrödinger operators, such as positivity, essential self-adjointness, and spectral properties, and to the study of a class of nonlinear Schrödinger equations with critical power-nonlinearities and multi-singular anisotropic inverse square potentials. Conditions on strength, location, and orientation of singularities are given for the minimum of the associated Rayleigh quotient to be achieved, both in the whole $\mathbb{R}^N$ and in bounded domains.

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Curved interface solution to the perturbed two-dimensional Allen-Cahn equations

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T. Kolokolnikov, J. Wei and J. Rumsey

We consider equilibrium solutions to a perturbed Allen-Cahn model in bounded 2-dimensional domains that have the form of a curved interface. Using singular perturbation techniques, we fully characterize the stability of such an equilibrium in terms of a certain geometric eigenvalue problem, and give a simple geometric interpretation of our stability results. Full numerical computations of the associated two-dimensional eigenvalue problem are shown to be in excellent agreement with the analytical predictions.

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Stochastic Stokes' Drift: Diffuse travelling fronts.

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The work described in this talk is motivated by the study of large time asymptotics of Brownian ratchets in unbounded domains, with applications to molecular motors in biology, and the Stochastic Stokes' drift, a phenomenon in which particles suspended in a liquid and subject to diffusion experience a net drift due to a wave traveling through the liquid.

Using the functional inequalities we show that when a traveling periodic potential is coupled to a diffusion, the solution of the relevant parabolic problem becomes a traveling diffused front, which behaves at large scale and in the reference frame attached to the center of mass as a solution of the heat equation with a modified diffusion coefficient, modulated by a periodic perturbation with fast oscillations.

We derive a formal asymptotic expansion of the traveling diffusion front corresponding to the stochastic Stokes' drift with given potential flow, and justify it rigorously. We use logarithmic Sobolev inequalities to obtain explicit rates of convergence.

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On the resonance condition of a singular limit problem from the Ohta-Kawasaki and the Gierer-Meinhardt theories

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The resonance condition, also known as the gap condition, has appeared in several recently studied singular perturbation problems. One natural question is whether such a condition is superfluous, and a better question is whether the condition implies a bifurcation phenomenon. In this talk I will discuss this condition in the case of a geometric problem that arises as a singular limit of the Ohta-Kawasaki theory of block copolymers and also of the Gierer-Meinhardt theory of morphogenesis. We show that in general standard bifurcation does not occur and the cause of resonance is typically an imperfect type of bifurcation.

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Self Replicating Spots for Reaction-Diffusion Systems in Two-Space Dimensions

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We analyze the dynamical behavior of multi-spot solutions in a two-dimensional domain Ω for certain two-component reaction-diffusion models in the singularly perturbed limit of small diffusivity ϵ for one of the two components. A formal asymptotic analysis, which has the effect of summing infinite logarithimic series in powers of $-1/\log\epsilon$, is used to derive an differential algebraic system of ODE's characterizing the slow dynamics of the spot locations. By numerically examining the stability thresholds for a single spot solution, a specific and simple criterion is formulated to theoretically predict the initiation spot-replication events. The analytical theory is compared with full numerical results.

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Spikes for the Gierer-Meinhardt system with Variable Coefficients

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We consider the Gierer-Meinhardt system with variable coefficients in the following two cases:

(i) Jump discontinuity for the diffusion coefficient of the inhibitor. (ii) Variable coefficient of the linear term in the activator equation (precursor inhomogeneity).

We prove results on the existence and stability of multiple spikes. A common phenomenon for variable coefficients is the existence of stable asymmetric spikes, which have varying amplitudes and irregular spacing. Particular effects include

(i) Existence of a spike near the jump discontinuity of the inhibitor. (ii) A spike can be located in certain subdomains, but not in others (localization principle). (iii) Destabilisation of multiple spikes by the precursor inhomogeneity.

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