Special Session 48: Inverse Problems and Imaging

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Small-energy limits of the matrix Schrödinger equation on the half line

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In this joint work with M. Klaus and R. Weder, the matrix Schrödinger equation on the half line is considered with the general selfadjoint boundary conditions at the origin. The small-energy limits of the scattering coefficients are analyzed.

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Recent Developments on the Qualitative Methods Inverse Elctromagntic Scattering

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Inverse scattering theory has been a particularly active area in applied mathematics for the past twenty five years. The aim of research in this field has been to not only detect but also to identify unknown objects through the use of acoustic, electromagnetic or elastic waves. Mathematically, such problems lead to nonlinear and severely ill-posed equations. Until a few years ago, essentially all existing algorithms for target identification were based on either a weak scattering approximation or on the use of nonlinear optimization techniques. In recent years alternative methods for imaging have been developed which avoid incorrect model assumptions inherent in weak scattering approximations and, as oppose to nonlinear optimization techniques, do not require a priori information. Such methods come under the general title of qualitative methods in inverse scattering theory.

This lecture will provide a review of the state of art of qualitative methods in inverse electromagnetic scattering theory. Numerical examples showing the practicality of these approaches to the problem of target identification will be given and finally some related open problems will be discussed.

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A Functional FIO Calculus for Linearized Offshore Seismic Imaging

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Under fairly general conditions, the forward scattering map in linearized seismic imaging is known to be a Fourier integral operator (FIO). However, the presence of caustics in the ray geometry of the smooth background soundspeed often lead to FIOs which lie outside the standard FIO calculus. We describe the geometry and microlocal analysis that arise for the marine data acquisition geometry in the presence of caustics of fold type.

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Convex source support and its application to electric impedance tomography

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The aim in electric impedance tomography is to recover the conductivity inside a physical body from boundary measurements of current and voltage. In many situations of practical importance, the investigated object has known background conductivity but is contaminated by inhomogeneities. In this work, we try to extract all possible information about the support of such inclusions inside a two-dimensional object from only one pair of measurements of impedance tomography. Our noniterative and computationally cheap method is based on the concept of convex source support, which stems from earlier works of Kusiak and Sylvester. The functionality of our algorithm is demonstrated by various numerical experiments.

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Rigidity of broken geodesic flow and inverse problems for radiative transfer equation

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Consider a broken geodesics $\alpha([0,l])$ on a compact Riemannian manifold (M,g) with boundary of dimension $n \geq 3$. The broken geodesics are unions of two geodesics with the property that they have a common end point. Assume that for every broken geodesic $\alpha([0,l])$ starting at and ending to the boundary ∂M we know the starting point and direction $(\alpha(0),\alpha'(0))$, the end point and direction $(\alpha(l),\alpha'(l))$, and the length l. We show that this data determines uniquely, up to an isometry, the manifold (M,g). This result has applications in inverse problems on very heterogeneous media for situations where there are many scattering points in the medium, and arises in several applications including geophysics and medical imaging. As an example we consider the inverse problem for the radiative transfer equation (or the linear transport equation) with a non-constant wave speed. Assuming that the scattering kernel is everywhere positive, we show that the boundary measurements determine the wave speed inside the domain up to an isometry.

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Regularized solutions to the inverse conductivity problem by the D-bar method

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A strategy for regularizing the inversion procedure for the 2-D D-bar reconstruction algorithm for electrical impedance tomography is presented. The strategy utilizes truncation of the boundary integral equation and the scattering transform. It is shown that this leads to a bound on the error in the scattering transform and a stable reconstruction of the conductivity. Results of the D-bar algorithm on simulated and experimental data are included.

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Numerical Studies on an Inverse Elliptic Problem for Optical Tomography

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A new numerical method for an inverse problem for an elliptic equation with unknown coefficient is purposed. In this problem the constant wave for probing light scattering media is running along a straight line and the source-dependent Dirichlet boundary condition is measured as the data for the inverse problem. After we transform the original inverse problem into an integral-differential equation, we has purposed the new method for the approximation of tail-function, which appears due to the truncation of an improper integral with respect to the source position, in order to solve the problem. Numerical experiments in the 2-D case are presented.

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Carleman estimates and anisotropic inverse problems

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We consider the imaging of anisotropic materials by electrical measurements. This inverse problem arises in Electrical Impedance Tomography (EIT), which has been proposed as a diagnostic method in medical imaging and nondestructive testing. The mathematical model is the anisotropic Calderón problem, which consists in determining a matrix of coefficients in an elliptic equation from boundary measurements of solutions.

The anisotropic Calderón problem can be formulated in geometric terms, as the recovery of a Riemannian metric from Cauchy data of harmonic functions on a manifold. Our approach is based on Carleman estimates. We characterize those Riemannian manifolds which admit a special limiting Carleman weight. By using these weights, we construct special harmonic functions and prove uniqueness results in anisotropic inverse problems for a class of Riemannian manifolds.

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Direct reconstruction method for electrical impedance tomography

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Kari Astala, Jennifer Mueller and Lassi Paivarinta

The uniqueness theorem for general conductivities by Astala and Päivärinta [Ann.Math.163(2006)] has a constructive proof. A practical EIT algorithm based on this theory is discussed. Complex geometrical optics solutions for the Beltrami equation play a key role in the proof. Numerical computation of those solutions is demonstrated. Furthermore, reconstructions from noisy data are presented.

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Cloaking and Transformation Optics

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Allan Greenleaf, Yaroslav Kurylev and Matti Lassas

We describe recent progress on making objects invisible to electromagnetic, acoustic and other types of waves.

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