Special Session 49: Recent developments of analytic and algebraic methods in integrable systems and applications

Tuncay Aktosun, University of Texas at Arlington, USA
Sarbarish Chakravarty, University of Colorado at Colorado Springs, USA
Kenichi Maruno, The University of Texas-Pan American, USA

Exact solutions to the sine-Gordon equation

Mehmet Akinlar
University of Texas at Arlington, USA
akinlar@uta.edu

A systematic method is presented to obtain certain solutions to the sine-Gordon equation. Such solutions are expressed in terms of algebraic combinations of polynomial, exponential, and trigonometric functions of the temporal and spatial variables. These explicit solutions are displayed in a simple compact form in terms of a square matrix A, a constant row vector C, and a constant column vector B, where A appears in a matrix exponential. Some illustrative examples with Mathematica animations are provided.

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Time evolution of the scattering data for an integrable system associated with a fourth-order ODE

Tuncay Aktosun
University of Texas at Arlington, USA
aktosun@uta.edu
Vassilis G. Papanicolaou

An integrable system of coupled nonlinear PDEs is considered, which is associated with the fourth-order linear ordinary differential operator $D^4 + DuD + v$ containing two potentials u and v. The time evolution of the scattering data is obtained with the help of the corresponding Lax pair.

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Group-invariant soliton equations and bi-Hamiltonian flows in symmetric Lie algebras

Stephen Anco
Departtment of Mathematics, Brock University (Canada), Canada
sanco@brocku.ca

This talk will present an algebraic derivation of soliton equations and their bi-Hamiltonian integrability structure from curve flows in symmetric Lie algebras. The derivation uses an algebraic construction of moving parallel frames in such Lie algebras and leads to a wide class of bi-Hamiltonian soliton equations describing multicomponent group-invariant integrable systems of mKdV type, NLS type, and sine-Gordon type. Corresponding to these soliton equations are Lie-algebraic variants of integrable vector models, such as the O(n) Heisenberg model and O(n) sigma model.

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Soliton solutions of the Davey-Stewartson equation

Gino Biondini
State Univerity of New York at Buffalo, USA
biondini@buffalo.edu
Ken-ichi Maruno and Sarbarish Chakravarty

Soliton solutions of (2+1)-dimensional integrable systems have received renewed interest in recent years, and several works have shown that they describe a large variety of soliton interactions. Among them is the phenomenon of soliton resonance, first derived in the context the Kadomtsev-Petviashvili (KP) equation. Recently, more general solutions of KP, possessing a web-like structure, have been studied, and similar solutions have been found for other discrete and continuous integrable systems. Indeed, it was conjectured that soliton resonance and web-like structure are a generic feature of (2+1)-dimensional integrable systems whose solutions can be expressed in determinant form.

In this talk we characterize a large class of soliton solutions of the Davey-Stewartson (DS) equation, and we study their asymptotics and interaction patterns in the xy-plane. Many of these solutions describe phenomena of soliton resonance and web structure. In particular, a subclass of solutions of DS is the analog of the soliton solutions of the Kadomtsev-Petviashvili II equation. In addition to these, however, we show that more general solutions exist, which describe resonant phenomena that have no counterpart in the KP equation, including the so-called phenomenon of soliton reconnection.

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Intrinsic Formulation of Geometric Integrability and Generation of Conservation Laws

Paul Bracken
University of Texas, USA
bracken@panam.edu

An intrinsic version of the integrability theorem for the classical Backlund Theorem is presented. It is characterized by a one-form which can be put in the form of a Riccati system. It is shown how this system can be linearized and how an infinite number of conservation laws can be obtained.

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Asymptotics of the Semiclassical Sine-Gordon Equation

Robert Buckingham
University of Michigan Ann Arbor, USA
robbiejb@umich.edu
Peter D. Miller

The semiclassical limit of solutions to the sine-Gordon equation, where the initial condition is fixed and dispersion tends to zero, is motivated by the modeling of magnetic flux propagation in Josephson junctions. We discuss the recent discovery of two families of initial data, with topological charge zero and one, respectively, for which the sine-Gordon equation can be solved explicitly for arb itrarily small dispersion. Plots of the solutions for small dispersion reveal, depending on the choice of other parameters, regions of pure librational and rotational motion, as well as regions of multi-phase waves separated by primary and secondary nonlinear caustics. We present current progress on the asymptotic analysis of these solutions for small dispersion.

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Generalized Inverse Scattering Transform for the Nonlinear Schrödinger Equation

Theresa Busse
University of Texas at Arlington, USA
tbusse@uta.edu

In 1972 Zakharov and Shabat introduced a powerful method known as the inverse scattering transform to solve the initial-value problem for the nonlinear Schrödinger (NLS) equation. Due to mathematical and technical difficulties, this method has been available mainly in the case where the multiplicity of each bound state is one. In our research we generalize the inverse scattering transform for the NLS equation to the case where the multiplicity of each bound state is arbitrarily chosen.

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Integrable systems and modular forms

Sarbarish Chakravarty
University of Colorado at Colorado Springs, USA
chuck@math.uccs.edu

In this talk a special class of nonlinear integrable ODEs will be discussed. These equations admit modular forms as their special solutions, and their general solutions have natural boundaries in the complex plane.

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On the Convergence of Hill's Method

Christopher Curtis
University of Washington, USA
curtchr@amath.washington.edu
Bernard Deconinck

Hill's method is used to numerically approximate spectra of linear operators associated with, for instance, forward scattering problems and stability problems for nonlinear waves. In this talk, I will discuss results on the convergence of Hill's method, the rate of convergence, and the convergence of the associated eigenfunctions.

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Explicit Solutions of the Cubic Matrix Nonlinear Schrodinger Equation

Francesco Demontis
University of Cagliari, Italy
fdemontis@unica.it
Cornelis van der Mee

We derive a class of explicit global (in position and time) solutions of the focusing matrix nonlinear Schrodinger equation using straightforward linear algebra techniques involving matrix exponentials and algebraic Riccati equations. Two methods to generate these solutions are discussed. The first method departs from a matrix exponential representation of the Marchenko integral kernel and arrives at the NLS solution by applying the inverse scattering transform method. The second method consists of subsituting the solution formula obtained in the matrix NLS equation and observing that the substitution allows for increased generality. All well-known multisoliton solutions are reproduced, as well as various multipole solutions.

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The Semiclassical Modified Nonlinear Schrodinger Equation

Jeffery Difranco
Seattle University, USA
difranco@seattleu.edu

We study an integrable modification of the focusing nonlinear Schrödinger equation from the point of view of semiclassical asymptotics. We analyze the associated spectral problem and obtain bounds for the discrete spectrum, generalizing known results for the spectrum of the nonselfadjoint Zakharov-Shabat spectral problem. Additionally, we solve the this spectral problem in terms of special functions for a multiparameter family of initial data for all values of the semiclassical parameter. These results are viewed as part of an ongoing project analyzing the semiclassical asymptotics of modified nonlinear Schrodinger equation using the steepest descent techniques for oscillatory Riemann-Hilbert problems first developed by Deift and Zhou.

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Peakons with cubic nonlinearity

Andrew Hone
University of Kent, England
anwh@kent.ac.uk
Jing Ping Wang

We consider a new partial differential equation discovered by Vladimir Novikov in a symmetry classification of integrable analogues of the Camassa-Holm equation with cubic (rather than quadratic) nonlinearity. A matrix Lax pair and bi-Hamiltonian structure is presented, together with a reciprocal transformation to a negative flow in the Kupershmidt-Fordy-Gibbons hierarchy. The integrable dynamics of peakon solutions is derived by restricting one of the Poisson structures to the peakon submanifold, with the interaction of two peakons being solved explicitly.

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Invariants of certain parabolic subgroups and k-chop integrals of the full Kostant-Toda lattice

Kaoru Ikeda
Keio University, Japan
ikeda@hc.cc.keio.ac.jp

It is known that the full Kostant-Toda lattice has the integrals defined by the coefficients of the characteristic polynomial $\det(\lambda - X)$. Flaschka et al find new integrals so-called k-chop integrals. They show that they are invariants on some parabolic subgroups. In this talk, I show that k-chop integrals are also invariants on certain extended parabolic subgroups and consider the full Kostant-Toda lattice from the point of view of the pre homogeneous space.

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Enumerative combinatorics on discrete-time integrable systems

Shuhei Kamioka
Kyoto University, Japan
kamioka@amp.i.kyoto-u.ac.jp

A combinatorial interpretation of discrete-time integrable systems is presented, in which it is shown that the time-evolution recurrences of the discrete-time Toda molecule, relativistic Toda molecule and semi-infinite Lotka-Volterra equation are equivalent to equalities of weight of certain directed graphs. In particular, the dependent variables of the discrete-time integrable systems are regarded as labels of edges of the graphs, and then the time-evolution recurrence of them are done as such equalities of labels that bijections between paths on the graphs conserve weight of paths.

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Integrable systems and cohomology of certain real varieties

Yuji Kodama
The Ohio State University, USA
kodama@math.ohio-state.edu

I will show that there exists a connection between singular structure of integrable systems (such as KP

and Toda) and cohomology of real flag varieties (G/P for real split group G with parabolic subgroup P).

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Stability Analysis of Persisting Periodic Solutions to a Complex Ginzburg-Landau Perturbation of NLS

Stephane Lafortune
College of Charleston, USA
lafortunes@cofc.edu
Tom Ivey

It was shown in [Cruz-Pacheco, Levermore, and Luce (2004)] that a certain class of periodic solutions to the nonlinear Schrodinger equation (NLS) persist when the NLS is subject to a perturbation leading to the Complex Ginzburg Landau equation (CGL). In this presentation, I will show how one can use methods coming from the theory of integrability together with the Evans function to study the spectral stability of these persisting solutions. In particular we show that the solutions of NLS are spectrally stable with respect to periodic perturbations. However, the solutions can become unstable when NLS is perturbed to CGL.

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Wronskian and Casoratian determinant solutions to soliton equations

Wen-xiu Ma
University of South Florida, USA
mawx@cas.usf.edu

We will show how to use Wronskian and Casoratian determinants to solve soliton equations. The primary steps are to apply Hirota's bilinear forms and the Pluecker relations. Illustrative examples contain the celebrated Korteweg-de Vries equation, the Boussinesq equation, the Toda lattice equation and the Volterra lattice equation. Various solutions such as solitons, positons and complexitons are constructed from their corresponding Hirota's bilinear equations.

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Soliton solutions of 2-dimensional vector soliton systems

Kenichi Maruno
University of Texas-Pan American, USA
kmaruno@utpa.edu
Yasuhiro Ohta, Masayuki Oikawa

Soliton solutions of 2-dimensional vector soliton systems are presented. The detail of vector soliton interactions is discussed analytically. A nonlocal extension of those systems is also discussed.

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A stochastic traffic cellular automaton model with higher velocity

Junta Matsukidaira
Ryukoku University, Japan
junta@rins.ryukoku.ac.jp

In the first part of this talk, we briefly review the ultradiscretization (UD) methodand show how it works well in the analysis of traffic models including the ultradiscrete Burgers equation. After that, we propose the stochastic Fukui-Ishibashi model,which is the higher velocity extension of the stochastic ultradiscrete Burgers equa-tion, and analyze the relation between density and flow of traffics using the two cluster approximation.

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On the Semiclassical Limit for the Sine-Gordon Equation

Peter Miller
University of Michigan, USA
millerpd@umich.edu
Robert Buckingham

The Cauchy initial-value problem for the sine-Gordon equation in laboratory coordinates arises in many physical problems. For example, the idealized propagation of magnetic flux quanta in a long superconducting Josephson junction is described by this mathematical problem. Moreover, in this context, the disparity between laboratory scales and quantum scales suggests a semiclassical scaling in which a typical initial disturbance liberates an enormous number of flux quanta. We will describe an ongoing research project to apply modern methods of asymptotic analysis for Riemann-Hilbert problems to calculate precise semiclassical asymptotics in this situation.

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Bidifferential graded algebras and integrable models

Folkert Mueller-hoissen
MPI for Dynamics and Selforganization, Goettingen, Germany
folkert.mueller-hoissen@ds.mpg.de
Aristophanes Dimakis

A bidifferential graded algebra (or "bidifferential calculus") is a graded algebra with respect to two different derivations which anticommute. Many integrable systems can be expressed in terms of this structure [1,2]. In fact, it is a particular case of a zero curvature condition, a familiar structure related to integrability. A bidifferential graded algebra generalizes the structure underlying the (anti-) selfdual Yang-Mills system, but moreover also covers e.g. the Kadomtsev-Petviashvili hierarchy. An appealing aspect of this formalism lies in the fact that it generates universal formulae for various algebraic aspects of integrability, like Bäcklund and Darboux transformations, and other solution generating methods. Here we present some new results and in particular a general and simple method to construct exact solutions.

1 A. Dimakis and F. Müller-Hoissen, *J. Phys. A: Math. Gen.* **33** (2000) 957, **33** (2000) 6579, **34** (2001) 9163
2 A. Dimakis and F. Müller-Hoissen, arXiv:0706.1373

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Darboux transformations for noncommutative systems.

Jonathan Nimmo
University of Glasgow, Scotland
j.nimmo@maths.gla.ac.uk
Jonathan J C Nimmo

This talk will review some recent applications [2,3,4] of quasideterminants [1] to the use of Darboux transformations to construct exact solutions to noncommutative versions of a number of classical integrable systems.

1. Gelfand, I. M. and Retakh, V. S. Funktsional. Anal. i Prilozhen., 25, 13 (1991).
2. Gilson, C. R. and Nimmo, J. J. C. J. Phys. A, 40, 3839 (2007).
3. Gilson, C. R., and Sooman, C. M. J. Phys. A, 41 (2008).
4. Li, C. X. and Nimmo, J. J. C. Proc. R. Soc. Lond. 646, 951 (2008).

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Classification of integrable Camassa-Holm type equations

Vladimir Novikov
Loughborough University, UK, England
V.Novikov@lboro.ac.uk
A.N.W. Hone and J.P. Wang

We classify generalisations of the Camassa-Holm and Degasperis-Procesi equations, which possess infinite hierarchies of quasi-local higher symmetries. We present the Lax representations, bi-Hamiltonian structures and recursion operators for these equations. We also study peakon-type solutions of some new equations, obtained in the result of the classification.

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Two-dimensional Interaction of Intermediate Long Solitary Waves

Masayuki Oikawa
Research Institute for Applied Mechanics, Kyushu University (RIAM, Kyushu Univ.), Japan
oikawa@riam.kyushu-u.ac.jp
Hidekazu Tsuji

In this work we investigate two-dimensional interaction of intermediate long solitary waves in a two-layer fluid model. First, for weak nonlinear waves, we consider the weak interaction of two solitary waves by using a perturbation method. Next, we investigate the strong interaction of solitary waves by numerical computations of a two-dimensional intermediate long wave equation for initial and boundary conditions simulating the refrection of a solitary wave due to a rigid boundary.

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QR-type algorithms and lattices of Toda type

Virgil Pierce
The Ohio State University, USA
virgilpierce@gmail.com
Yuji Kodama

The QR-algorithm is a method for computing the eigenvalues of a symmetric matrix which centers around the QR-factorization routine. There is an intimate relationship between the QR-algorithm and the Toda lattice hierarchy of equations. The SR-algorithm is a QR-type algorithm for computing the eigenvalues of a symplectic matrix which centers around the SR-factorization routine. We have shown that there is also a connection between the SR-algorithm and the Pfaff lattice hierarchy. The Pfaff lattice hierarchy was introduced by Adler and van Moerbeke to describe the partition functions of GOE and GSE random matrices. This connection has important consequences for the behavior of the SR-algorithm.

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Inverse scattering transform for the vector NLS equation with non-vanishing boundary conditions

Barbara Prinari
Dipartimento di Fisica - Universita di Lecce, Italy
barbara.prinari@le.infn.it
M.J. Ablowitz, G. Biondini and A.D. Trubatch

The inverse scattering transform for the vector defocusing vector nonlinear Schrodinger NLS equation with non-vanishing boundary values at infinity is constructed. The direct scattering problem is formulated on a two-sheeted covering of the complex plane. In the two component case, two out of the six scattering eigenfunctions do not admit an analytic extension on either sheet of the surface. Two additional analytic solutions are constructed by considering *adjoint* eigenfunctions. The discrete spectrum, bound states and symmetries of the direct problem are discussed. In general a discrete eigenvalue corresponds to a quartet of zeros (poles) of certain scattering data. The inverse scattering problem is formulated in terms of a Riemann-Hilbert (RH) problem in the upper/lower half planes of a suitable uniformization variable. Special soliton solutions, which have dark solitonic behavior in both components and ones which have one dark and one bright component are constructed from the poles in the RH problem. The generalization to vector NLS with an arbitrary number of components will also be discussed.

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Discrete Integrable Systems

Reinout Quispel
La Trobe University, Australia
r.quispel@latrobe.edu.au

Continuous integrable systems have been known for many years. Examples include the Kepler problem and the Korteweg-de Vries equation. On the contrary, discrete integrable systems, while arguably more fundamental, have only come to the fore in the last 2 decades. We plan to give a brief survey of this new field, concentrating on recent results in the area of discrete integrable mappings (i.e.integrable ordinary difference equations) as well as integrable partial difference equations.

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On the time evolution of a relative reflection coefficient under the KdV flow

Alexei Rybkin
University of Alaska Fairbanks, USA
ffavr@uaf.edu

We are concerned with the KdV equation on the full-line with real non-decaying initial profiles. We introduce a reflection coefficient and find its time evolution under the KdV flow. Some other issues related the inverse scatteringag transform will also be discussed.

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Noncommutative Equations and Solutions of Integrable Systems

Cornelia Schiebold
Mid Sweden University, Sweden
cornelia.schiebold@miun.se

The starting point will be a general approach to integrable systems based on the pioneering work of Marchenko about operator equations and a brilliant discovery by B. Carl who established the link to the modern geometric analysis on Banach spaces. The idea is to find noncommutative translations for both a soliton equation and a parameter-dependent solution. In a second step one returns to the scalar or matix equation using the theory of traces and determinants on Banach ideals. The profit is that one obtains in the end solution formulas depending on operator parameters yielding a quite explicit access to very general families of solutions.

The first part becomes algebraically involved if one tries to apply it to infinite families of integrable systems. We will outline this for the AKNS system and the KdV hierarchy, presenting in the latter case recent joint research with Sandra Carillo, Rome. Then we will try to give an impression how to use this in the qualitative study of solutions in the case of the NLS. Our main application will be a complete asymptotic description of multi-pole solutions, where one obtains a very neat correspondence between spec-

tral data and the dynamics of the solution.

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Some Classes of Surfaces arising from Soliton Theory and a Variational Principle

Suleyman Tek
University of Arkansas at Little Rock, USA
sxtek@ualr.edu

We consider 2-surfaces in $\mathbb{R}^3$ arising from the modified Korteweg-de Vries (mKdV) equation. We construct these surfaces using Lie theory and Integrable equations theory, and a variational principle. mKdV surfaces contain Willmore-like and algebraic Weingarten surfaces. We give a method for determining the position vector (parametrization) of the mKdV surfaces explicitly for a given solution of mKdV equation.

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Reducibility to the canonical forms for nonautonomous soliton equations in higher-dimensions and their exact solutions

Kouichi Toda
Department of Mathematical Physics, Toyama Prefectural University, Japan
toda@pu-toyama.ac.jp

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Semiclassical Limit of Scattering Transform (Direct and Inverse) for the Focusing Nonlinear Schroedinger Equation.

Alexander Tovbis
University of Central Florida, USA
atovbis@pegasus.cc.ucf.edu

We discuss the progress in finding the semiclassical (zero dispersion) limit of the scattering transform for the focusing NLS, that put into correspondence the initial potential $q(x, \epsilon)$ and the scattering data. In the solitonless case, the scattering data consists of the reflection coefficient $r(z, \epsilon)$, where z is the spectral and ϵ is the semiclassical parameter respectively. So, in this case, we are considering the $\epsilon \to 0$ limit of the maps $q \mapsto r$ and $r \mapsto q$. General case is also discussed.

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Exact solutions of the potential derivative nonlinear Schrödinger equation

Takayuki Tsuchida
Okayama Institute for Quantum Physics, Japan
tutitaka_2008@mail.goo.ne.jp

One of the most important properties of the Kaup-Newell (derivative nonlinear Schrödinger) equation is that it allows the introduction of the potential variable. As a result, the elementary function solutions of the Kaup-Newell equation can be expressed as the partial x-derivatives of elementary functions. We propose a new set of formulas that can directly provide the solutions of (a vector/matrix generalization of) the potential Kaup-Newell hierarchy. The results can be generalized to the space-discrete case. This talk is based in part on the recent paper arXiv:0712.4373 [nlin.SI].

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Numerical Study for Two-dimensional Interaction of the Solitary Waves in Shallow Water System: Comparison with Web-like Solutions

Hidekazu Tsuji
RIAM, Kyushu Univ., Japan
tsuji@riam.kyushu-u.ac.jp
Masayuki Oikawa and Kenichi Maruno

Two-dimensional interaction of solitary waves in shallow water system is studied numerically. This system can be described by KP equation in case of weak nonlinearity and weak two-dimensionality. Concerning the KP equation, Biondini and Kodama recently proposed web-like solution. For the purpose of comparison with this interesting solutions, we investigate time development of solutions under various initial conditions.

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On the discrete coupled KP equation

Satoshi Tsujimoto
Kyoto University, Japan
tujimoto@i.kyoto-u.ac.jp

In this talk we discuss the discrete analogue of the coupled KP equation which is closely related to the skew-orthogonal polynomials. After the introduction of the spectral transformations of the skew-orthogonal polynomials by using the bilinear formalism, we give the nonlinear form of the discrete coupled KP equation.

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Matrix algebra techniques for solving the matrix KdV equation

Cornelis Van der mee
University of Cagliari, visiting the University of Texas at Arlington, USA
cornelis@krein.unica.it
Tuncay Aktosun and Francesco Demontis

We derive explicit solutions of the KdV and matrix KdV equations using matrix algebra techniques such as matrix exponentials and algebraic Riccati equations. Both the indirect method of following the "trajectory" of the inverse scattering transform and the direct method of simply substituting the solution obtained in the matrix KdV equation are presented. All known N-soliton solutions are derived.

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