Special Session 6: Global or/and Blowup Solutions for Nonlinear Parabolic Equations and Their Applications

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The effect of wind on the propagation of forest fires

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Wind is one of the main forces affecting the rate of forest fire spread. It may either increase or decrease the energy transferred toward the unburned areas. The present study is devoted to understanding the effect of wind on the propagation of forest fires. To model the propagation of forest fires a semi-physical approach is used. By this approach forest fire is considered as an adiabatic combustion process for solid fuel. In our study we analyze the behavior of combustion waves for different wind velocities. Three types of wind velocity with respect to burning region are considered. When the wind velocity is zero or the wind blows towards unburned region, the existence and uniqueness of the traveling wave is proven. However, the situation is much more interesting when the wind blows towards the burning region. In such a case, there might be no combustion waves, one combustion wave or even two combustion waves. In particular, two combustion waves are observed when the wind velocity is small. When the wind velocity is large, there are no combustion waves. In my talk I will discuss the approach for study of combustion wave problem and discuss the physical scenarios for obtained results.

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Regularity theory in Orlicz-Sobolev space for nonlinear parabolic equations with conormal boundary conditions

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We study a generalization of $W^{1,p}$-regularity theory to Orlicz-Sobolev space $W^{1,\phi}$ for a nonlinear conormal derivative problem with discontinuous nonlinearity in an irregular domain. Given a Young function ϕ we establish a global $W^{1,\phi}$-regularity for such a Neumann problem under an appropriate smallness condition on the nonlinearity and a geometric flatness condition on the boundary of the domain.

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Global and blowup solutions for quasilinear parabolic systems

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We study the global and blowup solutions for the degenerate and quasilinear parabolic systems with Dirichlet boundary conditions in a bounded domain. Under suitable conditions we show that whether the solutions are global or blowup depends only on the size of the domain. For some special cases the results are sharp. The key to the proof is an a priori estimate for a new functional.

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Strongly degenerate parabolic equations with saturating diffusion

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We first consider a nonlinear diffusion equation used to describe propagation of thermal waves in plasma or in a porous medium, endowed with a mechanism for flux saturation, which corrects the nonphysical gradient-flux relations at high gradients. We study the model both analytically and numerically, and discover that in certain cases the motion of the front is controlled by the saturation mechanism. Instead of the typical infinite gradients, resulting from the linear flux-gradients relations, we obtain a discontinuous front, typically associated with nonlinear hyperbolic phenomena. We prove that if the initial support is compact, independently of the smoothness of the initial datum inside the support, a shock discontinuity at the front forms in a finite time, and until then the front does not expand. Adding a nonlinear convection enhances the conditions for a breakdown. In fact,

the most interesting feature is the effect of criticality, that is, unlike small amplitude solutions that remain smooth at all times, large amplitude solutions may develop discontinuities. This feature is easily seen via the analysis of traveling waves: while small amplitude kinks are smooth, in large amplitude kinks part of the upstream-downstream transition must be accomplished via a discontinuous jump (subshocks). Thus induced discontinuities may persist indefinitely since the traveling waves represent a forced motion. Unlike the classical Burgers case, here, due to the saturation of the diffusion flux, the viscous forces have a bounded range. When the inertial forcing exceeds a certain threshold, the disparity between the inertial and dissipative forces is resolved by formation of a discontinuity.

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Total vs. single point blow-up phenomena for a nonlocal parabolic problem

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We consider the initial-boundary value problem of semilinear parabolic system with both local and localized reactions; $u_t = \Delta u + v^p(0,t) + u^r, v_t = \Delta v + u^q(0,t) + v^s$ in $B \times (0,T)$, $u(x,t) = v(x,t) = 0$ on $\partial B \times (0,T)$, $u(x,0) = u_0(x), v(x,0) = v_0(x)$ in B where $p, q, r, s > 1, B = \{x \in R : |x|$

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About a pseudo steady state invariant as a global attractor for class of the non-linear flows and application in engineering

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Motivated by the reservoir engineering concept of the well Productivity Index , authors introduced and analyzed a functional, denoted as diffusive capacity, for the solution of the initial boundary value problem IBVP for a linear parabolic equation. In the present paper similar features for non-linear flows in inhomogeneous porous media subjected to the Forchheimer equations are analyzed. It is shown that under some hydrodynamic and thermodynamic constraints there exists a so called pseudo-steady state regime for the Forchheimer flows in porous media. In other words, under some assumptions there exists a steady state invariant over a certain class of solutions to the transient IBVP modeling the Forchheimer flow for slightly compressible fluid. This invariant is the diffusive capacity, which serves as the mathematical representation of the so called well Productivity Index. It was shown that this invariant serves as a global attractor for two classes of IBVP with homogeneous boundary conditions and arbitral initial data. The obtained results enable computation of the well Productivity Index by resolving a single steady state boundary value problem for a second order quasi-linear elliptic equation. Analytical and numerical studies highlight some new relations for the well Productivity Index in linear and nonlinear cases.

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Potential comparison and L^1-convergence order for the PME and p-Laplacian equation

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We study an asymptotic L^1-convergence order for the PME and p-Laplacian equation with compactly supported radial symmetric initial data. The optimal asymptotic convergence order $O(1/t)$ will be obtained using a recently developed potential comparison method.

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Entropy dissipations methods for the sign-changing solutions of the heat equation with nolinear boundary condition

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We consider the sign-changing solution of the linear heat equation with nolinear boundary condition:$u_t = \Delta u$ in Ω,$-\partial u \partial x_N = |u|^{p-1}u$ on $\partial\Omega$, where Ω is a N-dimensional half space,$p > 1 + 1/N$.In this talk,we assume that the initial data is a L^1 and L^∞ function.Our aims are to study the asymptotic behavior of the L^1 bounded sign-changing solution of the this problem and to study the decay rate of the L^q distance.

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Potential comparison and asymptotics of equations in a divergence form

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Recently a potential comparison technique has been developed to obtain the optimal asymptotic convergence order $O(1/t)$ as $t \to \infty$ for a fast diffusion equation (see MR# 2246356). This method can be applied to other problems after a suitable modification. The goals of this talk are to demonstrate the potential comparison technique in a transparent way and improve the asymptotic convergence rate to problems of the type in the title. Nonlinear diffusions such as PME or p-Laplacian or a nonlinear convection are examples under consideration.

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The Hardy inequality and p-Laplace heat equation with singular potential on Carnot groups

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The Hardy inequality plays an important role in the study of partial differential equations. This inequality has been generalized and extended in various directions. In this talk we shall present a proof of Hardy's inequality on Carnot groups. We shall then investigate the nonexistence of positive solutions for the following nonlinear parabolic partial differential equation:

$$\frac{\partial u}{\partial t} = \Delta_{\mathbb{G},p} u + V(x) u^{p-1} \quad \text{in} \quad \Omega \times (0,T), \quad 1$$

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A positivity preserving central-upwind scheme for chemotaxis and haptotaxis models

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I will present a new finite-volume method for a class of chemotaxis models and for a closely related haptotaxis model. In its simplest form, the chemotaxis model is described by a system of nonlinear PDEs: a convection-diffusion equation for the cell density coupled with a reaction-diffusion equation for the chemoattractant concentration. It is well-known that solutions of such systems may develop spiky structures or even blow up in finite time provided the total number of cells exceeds a certain threshold. This makes development of numerical methods for chemotaxis systems extremely delicate and challenging task. The first step in the derivation of the new method is made by adding an equation for the chemoattractant concentration gradient to the original system. The convective part of the resulting system is then of a mixed hyperbolic-elliptic type and therefore straightforward numerical methods for the studied system may be unstable. The proposed method is based on the application of the second-order central-upwind scheme, originally developed for hyperbolic systems of conservation laws in, to the extended system of PDEs. The proposed central-upwind scheme is positivity preserving, which is a very important stability property of the method. The scheme is applied to a number of two-dimensional problems including the most commonly used Keller-Segel chemotaxis model and its modern extensions as well as to a haptotaxis system modeling tumor invasion into surrounding healthy tissue.

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Stability of traveling wavefronts for time-delayed reaction-diffusion equations

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This talk is concerned with time-delayed reaction-diffusion equations. I focus on the study of traveling wavefronts, and prove the wavefronts to be stable time-asymptotically. I also carry out some numerical simulations which confirm our theoretical results. Finally, some open questions in this topic will be mentioned.

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Euler equations on the Virasoro group

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A Sobolev space approach to global solutions for semilinear parabolic equations

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For a class of nonautonomous, semilinear parabolic equations, we establish the existence of solutions in a semi-infinite cylinder $[0, \infty) \times \Omega$, where Ω is a bounded domain in R^n. The main tool is the topological degree for proper Fredholm maps of index zero. Solutions have a prescribed initial value and vanishing Dirichlet boundary conditions. The functional setting provides specific information about the decay of the solutions as t grows large. A general result is obtained, and leads to a class of examples featuring a uniformly elliptic operator and a dissipative nonlinearity.

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The critical blow-up exponent for the equation $u_t = \Delta u + V(x)u + a(x)u^p$ in the case that the potential V decays quadratically

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Consider positive solutions to the equation $u_t = \Delta u + V(x)u + a(x)u^p$ in R^d, $d \geq 2$, where $p > 1$, a is on the order $|x|^m$, $m \in (-\infty, \infty)$, and $V(x) \sim \frac{\gamma}{|x|^2}$, $\gamma \in (-\infty, \frac{d-2}{8})$, as $|x| \to \infty$. We calculate explicitly a critical exponent p^*, depending on p, m and γ, with the property that all solutions blow up in finite time if $1p^*$.

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Asymptotic behavior of global solutions of a supercritical semilinear heat equation

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We consider the Cauchy problem for a semilinear heat equation with supercritical power nonlinearity. To a large extent, the asymptotic behavior of global solutions as time approaches infinity is determined by the way their initial conditions decay at spatial infinity. We first review various possibilities of the asymptotic behavior, including convergence to a single steady state and quasiconvergence to ordered and unordered families of steady states. We shall then focus on the case when the initial condition decays anisotropically - it has different decay properties along different rays from the origin.

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Blow-up at space infinity for quasilinear parabolic equations

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We discuss blow-up at space infinity for quasilinear parabolic equations $u_t = \Delta u^m + u^p$, where $m > 0$ and $p > 1$ are real constants, which are often called forced porous medium and fast diffusion equations. We study nonnegative blow-up solutions whose blow-up time coincide with those of solutions of O.D.E. $v' = v^p$ with initial data $\|u_0\|_\infty$. We show that such a solution blows up only at space infinity and possesses *blow-up directions* and that they are completely characterized by behavior of initial data. Moreover, necessary and sufficient conditions on initial data for blow-up at *minimal blow-up time* are also investigated.

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Asymptotic behavior of solutions of a semilinear heat equation with localized reaction

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We consider nonnegative solutions to the Dirichlet problem of a semilinear heat equation with localized reaction in Ω: $u_t = \Delta u + f(u(x_0(t), t))$, where Ω is a smoothly bounded domain, $x_0(t)$ is a locally Hölder continuous function from $[0, \infty)$ into Ω and f satisfies $f(0) = f'(0) = 0$ and some blow-up condition. We show that if $x_0(t)$ remains in some compact subset in Ω as $t \to \infty$ then all global solutions are bounded in $\Omega \times (0, \infty)$ and if $x_0(t)$ approches the boundary of Ω as $t \to \infty$ then some unbounded global solution (infinite time blow-up solution) exists. These results are parts of our main results on the classification of all solutions.

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Blow-up at isolated singularities of nonlinear parabolic inequalities

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We discuss how the blow-up behavior at an isolated singularity of solutions of nonlinear parabolic inequalities depends on an exponent in the inequalities. In particular, we show that for certain exponents there exists an apriori bound on the rate of blow-up, and for other exponents there does not exist such a bound.

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Global well-posedness of a haptotaxis model with spatial diffusion and age structure

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A system of non-linear partial differential equations modeling tumor invasion into tissue is analyzed. The model takes into account cell motility and haptotaxis, that is, the directed migratory response of tumor cells to the extracellular environment. Individual cell processes are modeled according to cell age. The equation for the tumor cell density thus incorporates second-order (parabolic) terms representing diffusion and taxis as well as a first-order (hyperbolic) part due to cell aging. Global existence and uniqueness of non-negative solutions is shown.

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