

## Special Session 9: Variational Inequalities, Theory, Methods and Applications in the Engineering Sciences

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### Some methods for solving metrically regular inclusions

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F. Aragon Artacho and A.L. Dontchev

We present some methods for solving variational inclusions of the form  $T(x) \ni 0$ . Such inclusions serve as a general tool for describing and solving a vast array of mathematical problems in a unified manner; they are also natural models for problems in economics and engineering. Variational inclusions have been extensively studied in the monotone framework; we focus here on mappings  $T$  that enjoys metric regularity properties and study the convergence and the stability of our methods.

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### Scalarly compactness, $(S)_+$ -type conditions, variational inequalities and complementarity problems in Banach spaces

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The main goal of this talk is to present a topological solvability method applicable to the study of solvability of variational inequalities and of complementarity problems in reflexive Banach spaces. Our method is based on the notion of scalarly compact operator and on  $(S)_+$ -type conditions.

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### An elastic viscoplastic contact problem

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The contact between an elastic-viscoplastic body and a deformable obstacle is studied. The effect of damage, due to internal tension or compression

and caused by the opening and growth of micro-cracks and micro-cavities, is also considered. The variational formulation leads to a coupled system of evolutionary equations. An existence and uniqueness result is established by using approximate problems, Schauder' fixed point theorem and a comparison result.

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### Pairs of positive solutions for $p$ -Laplacian equations with combined nonlinearities

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N. S. Papageorgiou

We consider a nonlinear Dirichlet problem driven by the  $p$ -Laplacian differential operator, with a nonlinearity concave near the origin and a nonlinear perturbation of it. We look for multiple positive solutions. We consider two distinct cases. One when the perturbation is  $p$ -linear and resonant with respect to  $\lambda_1 > 0$ ,  $p$  the principal eigenvalue of  $p$ - $\Delta_p$ ,  $W$  at infinity, and the other when the perturbation is  $p$ -superlinear at infinity. In both cases we obtain two positive smooth solutions. The approach is variational, coupled with the method of upper-lower solutions and with suitable truncation techniques.

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### Variational analysis of static frictional contact problems for electro-elastic materials

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We consider a mathematical model which describes the static frictional contact between a piezoelectric body and an electrically conductive foundation. The material behavior is described with a nonlinear electroelastic constitutive law. Both the frictional contact and the electrical conductivity on the contact surface are modeled with subdifferential boundary conditions. We derive a variational formulation of the problem which is in the form of a system coupling two

hemivariational inequalities for the displacement and the electric potential fields, respectively. Then we prove the existence of a weak solution to the model and, under additional assumptions, its uniqueness. The proofs are based on recent results for inclusions of subdifferential type in Sobolev spaces.

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### Positive and multiple solutions for hemivariational inequalities

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We prove the existence of a positive solution, of a negative solution and of a sign-changing (nodal) solution for a Dirichlet eigenvalue problem involving the  $p$ -Laplacian and a nonsmooth potential. The problem can be formulated as a hemivariational inequality for which no symmetries conditions are assumed.

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### Some variants of the Secant-type methods for variational inclusions

**Alain Pietrus**

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The aim of this talk is to present some variants of

the Secant-type method for solving a variational inclusion in the form  $0 \in f(x) + G(x)$  where  $f$  is a single function and  $G$  is a set-valued map. In a first time of the talk, we recall the classical secant method and associated methods as for example the combination of Newton's method with a secant-type method. In a second part of the talk we will give some results about the convergence of the (Steffensen-type method) sequence  $(x_k)$  satisfying  $0 \in f(x_k) + [g_1(x_k), g_2(x_k); f](x_{k+1} - x_k) + G(x_{k+1})$  under center-Holder condition on the first order divided difference, where  $g_1$  and  $g_2$  are some continuous functions parameter. The main tool of this study is a well-known fixed point theorem for set-valued maps given by A.L. Dontchev and W.W. Hager.

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### Non-smooth dynamical systems: an overview

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We review some recent results on the Lyapounov's stability of non-smooth dynamical systems. We give some conditions ensuring the finite time stability of a first order non-smooth dynamical system. Some extensions to the case of a non-smooth Lyapounov function are also studied. The theoretical results are supported by examples in nonsmooth mechanics and electronics.

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