Due at the start of class on Tues Nov 11, 2003.

Answer the following questions in groups of three. Turn in one solution sheet per group. Write the names of your group's members at the top of the first page of your solution sheet.

1. The following is INCORRECT: \( \lim_{x \to 0^+} \left( \frac{1}{x^2} - \frac{1}{x} \right) = \infty - \infty = 0. \)

   The following, using L'Hôpital's Rule, is also INCORRECT:
   \[
   \lim_{x \to 0^+} \left( \frac{1}{x^2} - \frac{1}{x} \right) = \lim_{x \to 0^+} \left( \frac{1 - x}{x^2} \right) = \lim_{x \to 0^+} \left( \frac{-1}{2x} \right) = -\infty.
   \]

   The limit is, in fact, \(+\infty\). Find what is wrong with the previous arguments and give a correct argument to compute the limit.

2. For each of the following parts, find a function \( f \) that satisfies the conditions given in that part. Note that each part has many solutions.

   (a) \(
   \lim_{x \to 2} f(x) = 0 \quad \text{and} \quad f(2) \text{ has the form } 0/0.
   \)

   (b) \(
   \lim_{x \to -1} f(x) = 2 \quad \text{and} \quad f(-1) \text{ has the form } 0/0.
   \)

   (c) \(
   \lim_{x \to \pi} f(x) = \infty \quad \text{and} \quad f(\pi) \text{ has the form } 0 \cdot \infty.
   \)

   (d) \(
   \lim_{x \to 0} f(x) = 0 \quad \text{and} \quad f(0) \text{ has the form } 0 \cdot \infty.
   \)

   (e) \(
   \lim_{x \to 0} f(x) = 1 \quad \text{and} \quad f(0) \text{ has the form } 0 \cdot \infty.
   \)

   (f) \(
   \lim_{x \to -\infty} f(x) = 0 \quad \text{with form } 0/0.
   \)

   (g) \(
   \lim_{x \to -\infty} f(x) = \infty \quad \text{with form } 0/0.
   \)

   (h) \(
   \lim_{x \to -\infty} f(x) = 3 \quad \text{with form } 0/0.
   \)

   (i) \(
   \lim_{x \to -\infty} f(x) = e \quad \text{with form } 0^0.
   \)

   (j) \(
   \lim_{x \to -\infty} f(x) = 0 \quad \text{with form } 0^0.
   \)

   (k) \(
   \lim_{x \to -\infty} f(x) = 1 \quad \text{with form } 0^0.
   \)

   (l) \(
   \lim_{x \to 0} f(x) = 0 \quad \text{with form } \infty/\infty.
   \)

   (m) \(
   \lim_{x \to 0} f(x) = \infty \quad \text{with form } \infty/\infty.
   \)

   (n) \(
   \lim_{x \to 0} f(x) = 1 \quad \text{with form } \infty/\infty.
   \)

   (o) \(
   \lim_{x \to -\infty} f(x) = e \quad \text{with form } 0 \cdot \infty.
   \)

   (p) \(
   \lim_{x \to -\infty} f(x) = \infty \quad \text{with form } 0 \cdot \infty.
   \)

END