Due at the start of class on Tues Nov 18, 2003.

Answer the following questions in groups of three. Turn in one solution sheet per group. Write the names of your group’s members at the top of the first page of your solution sheet.

NOTE: this worksheet is VERY long – so allow enough time!!

1. A rectangular yard is to be constructed along the side of a house by erecting a fence on three sides, using the house as the fourth wall of the yard. Find the dimensions that produce the yard of greatest area if fencing, 20 meters long, is available. (Note: this question can be done without calculus!)

2. A box with an open top and a square base is to be built so that the height of the box plus the length of one of the sides is to be 8 meters. Find the EXACT dimensions for such a box that yield the maximal volume; justify your reasoning, showing all steps in your solution. (Note: this question is §4.1 material (e.g., Example 9 on Page 192), although it can be done via later methods. This question was on Midterm 2.)

3. A poster is to contain 108 cm$^2$ of printed matter, with margins of 6 cm each at the top and bottom and 2 cm on the sides, as shown below in Figure 1. What is the least cost to make the poster if it costs 5 cents/cm$^2$ to make the part consisting of printed matter and 1 cent/cm$^2$ to make the part consisting of the margins?

4. A certain window consists of an equilateral triangle on top of a rectangular window, as shown below in Figure 2. The rectangular part of the window is made of clear glass and transmits twice as much light per square foot as the triangular part, which is made of stained glass. If the entire window has a perimeter of 20 ft, find the dimensions (to the nearest ft) of the window that will admit the most light.

5. A Norman window consists of a rectangle with a semicircle mounted on top, as shown above in Figure 3. What are the dimensions of the Norman window with largest area with a fixed perimeter of $P$ meters?
6. A manufacturer can produce radios at a cost of $8 each, and estimates that if they are sold for \( x \) dollars each, then consumers will buy \( 35 - x \) radios a day.

(a) Find a formula for the profit made per day.

(b) At what price should the manufacturer sell the radios to maximize the profit per day?

7. A bus company will charter a bus that holds 50 people to groups of 35 or more. If a group contains exactly 35 people, each person pays $60. In larger groups, everyone’s fare is reduced by $1 for each person in excess of 35.

(a) What is the revenue if the bus is chartered to 35 people?

(b) What is the revenue if the bus is chartered to 36 people?

(c) What is the revenue if the bus is chartered to 37 people?

(d) Find a formula for the revenue in terms of the number of people chartering the bus.

(e) Determine the size of the group for which the bus company’s revenue will be greatest. (Note: you need to check your answer makes physical sense and that it is as correct as the physical constraints allow.)

8. A hotel finds it can rent 200 rooms a day if it charges $40 per room. For each $1 increase in rental rates, four fewer rooms will be rented per day. What room rate maximizes revenue?

9. The number of boxes of a certain breakfast cereal that are sold is given by \( 5000e^{-0.5p} \) boxes per month when the price is \( p \) dollars per box. Determine the price that will result in the greatest consumer expenditure. (Note: you first have to find a formula for consumer expenditure; i.e., a formula for the amount of money consumers will spend on the cereal.)

10. Suppose that when \( q \) boxes of a certain cereal are produced, the total manufacturing cost is \( 3q^2 + 5q + 75 \) cents. At what level of production will the average cost per box be smallest? (Note: you should first find a formula for average cost.)