Due at the start of class on Tues Sept 21, 2004.

Answer the following questions in groups of two or three. Turn in one solution sheet per group. Write the names of your group’s members at the top of the first page of your solution sheet. Write neatly and orderly – points will be deducted for messy work.

In questions 1-5, sketch the graph of a function \( f \) that satisfies the given conditions.

1. \( \lim_{x \to 5} f(x) \) exists, but \( f \) is not continuous at \( x = 5 \).

2. \( f \) is continuous on \((-\infty, 1)\) and on \((1, +\infty)\), but \( f \) is not continuous on \((-\infty, +\infty)\).

3. \( f \) has domain \([0, 5]\) and is continuous on \([0, 5)\), but is not continuous on \([0, 5]\).

4. \( f \) is continuous everywhere except at \( x = 5 \), at which point it is continuous from the right.

5. \( f \) is discontinuous at \( x = 5 \), and \( f(5) = 1 \); however, this function \( f \) is such that if \( f(5) = 1 \) were changed to \( f(5) = 0 \), then \( f \) would be continuous at \( x = 5 \).
In questions 6-8, determine whether or not the given function is continuous (& so also determine the domain of the function), and explain your reasoning.

6. Your exact height as a function of time.

7. The volume of a melting ice cube as a function of time.

8. The cost of a long-distance telephone call charged by the minute (viewed as a function of time).

9. Find constants $a$ and $b$ so that $f$ will be continuous for all real numbers $x$. Give a coherent mathematical argument to justify your values for $a$ and $b.$

$$f(x) = \begin{cases} \frac{\sin ax}{x} & \text{if } x < 0 \\ 8 & \text{if } x = 0 \\ 3x + b & \text{if } x > 0. \end{cases}$$

10. Define a function that has domain $\mathbb{R}$ but is continuous NOWHERE. (Hint: consider a piecewise function with many (& I mean many) pieces.)