Due at the start of class on Tues Nov 9, 2004. Answer the following questions in groups of two or three. Turn in one solution sheet per group. Write the names of your group’s members at the top of the first page of your solution sheet. Write neatly and orderly – points will be deducted for messy work.

1. Consider whether or not the following statement is true for all functions $f$ with the stated properties: if true for all such $f$, then explain why; if false for some $f$, then give an example of such an $f$ to support your claim (such an example is called a ‘counter-example’).
   If $f$ is continuous on $[-1, 1]$ and $f(-1) = -1$ and $f(1) = 1$, then there exists a number $c \in (-1, 1)$ such that $f(c) = 0$.

2. Consider whether or not the following statement is true for all functions $g$ with the stated properties: if true for all such $g$, then explain why; if false for some $g$, then give a counter-example.
   If $g$ is defined on $[-1, 1]$ and $g(-1) = -1$ and $g(1) = 1$, then there exists a number $c \in (-1, 1)$ such that $g(c) = 0$.

3. Consider whether or not the following statement is true for all functions $h$ with the stated property: if true for all such $h$, then explain why; if false for some $h$, then give a counter-example.
   If $h$ is continuous at $a$, then $h$ is differentiable at $a$.

4. Consider whether or not the following statement is true for all functions $F$ with the stated property: if true for all such $F$, then explain why; if false for some $F$, then give a counter-example.
   If $F$ is differentiable at $a$, then $F$ is continuous at $a$.

5. Sketch the graph of one function $f$ with all the following properties:

   $f'(x) < 0$ for $x < -1$,
   $f'(x) < 0$ for $x > 3$,
   $f'(x) > 0$ for $x \in (-1, 3)$,
   $f''(x) > 0$ for $x < 2$,
   $f''(x) < 0$ for $x > 2$.

   (Note: you are NOT asked to find a formula for $f$, and there could be more than one correct answer to this question, or there could be no such $f$.)

   If you claim that there is no such $f$, then you should justify your claim.

*=mandatory
6. Sketch the graph of one function $f$ with all the following properties:

- $f'(x) < 0$ for $x < 1$,  
- $f''(x) < 0$ for $x < 1$,  
- $f'(x) > 0$ for $x > 1$,  
- $f''(x) < 0$ for $x > 1$.

(Note: you are NOT asked to find a formula for $f$, and there could be more than one correct answer to this question, or there could be no such $f$.)

If you claim that there is no such $f$, then you should justify your claim.

What can you say, if anything, about the derivative of $f$ at $x = 1$?

7. Sketch the graph of one function $f$ with all the following properties:

- $f'(x) < 0$ for $x \in (-\infty, -6) \cup (2, \infty)$,  
- $f'(x) > 0$ for $x \in (-6, 2)$,  
- $f(-6) = 5$,  
- $f(2) = -1$.

(Note: you are NOT asked to find a formula for $f$, and there could be more than one correct answer to this question, or there could be no such $f$.)

If you claim that there is no such $f$, then you should justify your claim.

8. (a) Let $f(x) = Ax^2 + Bx + C$, where $A, B, C$ are constants and $A \neq 0$. Find values for $A$, $B$ and $C$ such that $f(3) = 0$, $f'(x) < 0$ for $x < 1$ and $f'(x) > 0$ for $x > 1$. (Note: there could be more than one correct answer to this question.) Check that your $f$ satisfies the stated properties.

(b) Redo (a) but this time find all possible values of $A, B$ and $C$ that give $f$ the required properties; justify your claim.

(c) Find a quadratic polynomial $g$ that satisfies $g(0) = g'(0) = g''(0) = 3$, and check that your $g$ satisfies the stated properties.