Due at the start of class on Tues Nov 16, 2004. Answer the following questions in groups of two or three. Turn in one solution sheet per group. Write the names of your group’s members at the top of the first page of your solution sheet. Write neatly and orderly – points will be deducted for messy work.

1. The following is INCORRECT: \[ \lim_{x \to 0^+} \left( \frac{1}{x^2} - \frac{1}{x} \right) = \infty - \infty = 0. \]

The following, using L’Hôpital’s Rule, is also INCORRECT:
\[ \lim_{x \to 0^+} \left( \frac{1}{x^2} - \frac{1}{x} \right) = \lim_{x \to 0^+} \left( \frac{1-x}{x^2} \right) = \lim_{x \to 0^+} \left( \frac{-1}{2x} \right) = -\infty. \]

The limit is, in fact, +\(\infty\). Find what is wrong with the previous arguments and give a correct argument to compute the limit.

2. Can one use L’Hôpital’s Rule to find \( \lim_{x \to 0} x^2 \sin(1/x) \)? Explain and find the limit.

3. Let \( f(x) = \sqrt{x^2 - x} - x \).
   (a) Writing \( f(x) = \frac{x^{-1} (\sqrt{x^2 - x} - x)}{x^{-1}} = \frac{\sqrt{1 - \frac{1}{x} - 1}}{x^{-1}} \), use L’Hôpital’s Rule to find \( \lim_{x \to \infty} f(x) \).
   (b) Instead, writing \( f(x) = \frac{\sqrt{x^2 - x} - x}{1} = \frac{(\sqrt{x^2 - x} - x)(\sqrt{x^2 - x} + x)}{\sqrt{x^2 - x} + x} = \frac{-x}{\sqrt{x^2 - x} + x} \), withOUT using L’Hôpital’s Rule, find \( \lim_{x \to \infty} f(x) \). Check that your answer matches your answer from (a).

4. (a) What is the logarithmic rule for \( \ln x^p \)?
   (b) Find \( \ln x^{a/(\ln x)} \).
   (c) We will now see that a limit of type \( 0^\infty \) or \( \infty^0 \) can be any value. Let \( a \in \mathbb{R} \) be a nonzero constant, and let \( c \) denote \( 0^+ \) or \( \infty \). Write \( L = \lim_{x \to c} x^{a/(\ln x)} \). Using (b), find \( L \).
   (d) In (c), what value should \( a \) be to force \( L = 2 \)?
   (e) In (c), what value should \( a \) be to force \( L = 5 \)?
   (f) Write down your favorite number. In (c), what value should \( a \) be to force \( L = \) your favorite number?

* = mandatory

OVER
5. A rectangle is bounded by the $x$-axis, the $y$-axis and the graph of $y = (6 - x)/2$. What length and width should the rectangle have so that its area is a maximum?

6. A right triangle is formed in the first quadrant by the $x$-axis, the $y$-axis and a line through the point (1, 2).
   
   (a) Write the height of the triangle as a function of its width along the $x$-axis.
   
   (b) Find the vertices of the triangle such that its area is a minimum.

7. A solid is formed by adjoining two hemispheres to the ends of a right circular cylinder (one hemisphere on each end). The total volume of the solid is 12 cubic centimeters. Find the radius of the cylinder that produces the minimum surface area. (A sphere of radius $r$ has volume $\frac{4}{3}\pi r^3$ and surface area $4\pi r^2$.)

8. A piece of wire 30 cm long is cut into two pieces; one piece is bent into a square and the other piece is shaped into a circle. Assuming the wire is cut so that the total area enclosed by the square and circle is a maximum, how long should be the piece that is cut to form the square?