**Study Technique 4**  Try this technique to help you prepare for a test. A couple/few days before the test, look over all the homework. The day/night before the test, look over the homework again, and ask yourself what the key concepts and methods are per question (WITHOUT reworking the questions). Jot this down on a sheet of paper and, when you are done, compare with your solutions to see if you were correct. Any question in which you were on the wrong track, look over your solution to it to make sure you see what the main idea is. This work the day/night before will help get your brain to think faster and bring the material to the front of your brain to help you think faster during the actual test.

These questions are not due. Test 2 will be Wed Oct 24 during lab time.

Below are practice questions for Test 2 (in no particular order). Test 2 will cover college algebra and material in Secs 2.1-3.9 inclusive. The homework is available from my website: www.uta.edu/math/vancliff/T/F07.

1. Rework the practice questions provided for Test 1 on worksheet 3, as well as Test 1.

2. Go over all assigned homework, worksheet questions and quizzes.

3. Functions $f$ and $g$ and their first and second derivatives, $f'$, $g'$, $f''$, $g''$, are defined on $\mathbb{R}$, and, at 0, 1, 2, they take on the values given in the following table.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x)$</th>
<th>$g(x)$</th>
<th>$f'(x)$</th>
<th>$g'(x)$</th>
<th>$f''(x)$</th>
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<tbody>
<tr>
<td>0</td>
<td>5</td>
<td>3</td>
<td>2</td>
<td>-10</td>
<td>7</td>
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<td>1</td>
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</table>

(a) Let $G(x) = x^4 f(x)$. Compute $G'(2)$ if it exists; if it does not exist, explain why not.

(b) Let $H(x) = (g \circ f)(x) = g(f(x))$. Compute $H'(1)$ if it exists; if it does not exist, explain why not.

(c) Let $F(x) = e^{g(x)}$. Compute $F'(0)$ if it exists; if it does not exist, explain why not.

(d) Let $F(x) = e^{g(x)}$. Compute $F''(0)$ if it exists; if it does not exist, explain why not.

(e) Let $J(x) = \sin(x + g(x) - 3)$. Compute $J'(0)$ if it exists; if it does not exist, explain why not.

(f) Let $K(x) = \ln f(x)$. Compute $K'(0)$ if it exists; if it does not exist, explain why not.

(g) Let $M(x) = \ln f(x)$. Compute $M'(2)$ if it exists; if it does not exist, explain why not.

4. The functions $f$ and $g$ and their first and second derivatives, $f'$, $g'$, $f''$, $g''$, are defined on $\mathbb{R}$, and, at 0, 1, 2, they take on the values given in the following table.

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</tbody>
</table>
(a) If \( F(x) = \frac{f(x)}{x} \), compute \( F'(2) \).

(b) If \( G(x) = (g(x))^3 \), compute \( G''(2) \).

(c) If \( H(x) = f(x)e^{3x} \), compute \( H'(0) \).

(d) If \( K(x) = \tan^{-1}(f(x)) = \arctan(f(x)) \), compute \( K'(1) \).

5. Some functions and their corresponding derivatives are shown. Give the steps needed to compute the derivatives (NOT by using a limit) and state which rules you use.

(a) \( f(x) = e^{\sin x} \)  \( \Rightarrow f'(x) = (\cos x)e^{\sin x} \)

(b) \( f(x) = \frac{5x + 2}{\tan x} \)  \( \Rightarrow f'(x) = \frac{5 \cos x \sin x - 5x - 2}{7 \sin^2 x} \)

(c) \( f(x) = 2x^2 \)  \( \Rightarrow f'(x) = (\ln 2)2x^{2+1} \)

(d) \( 4 \cos x \sin y = e^y \)  \( \Rightarrow \frac{dy}{dx} = \frac{4 \sin x \sin y}{4 \cos x \cos y - e^y} \)

6. Given that \( x^y = y^x \), compute \( \frac{dy}{dx} \) at the point \( (7, 7) \).

7. Given that \( x \cos y = y \sin x \) and that \( \cos \frac{\pi}{4} = \frac{1}{\sqrt{2}} = \sin \frac{\pi}{4} \), compute \( y' \) at the point \( \left( \frac{\pi}{4}, \frac{\pi}{4} \right) \).

(a) 0.681  \( (b) 1.468 \)  \( (c) \frac{4 + \pi \ln \pi - \pi \ln 4}{4 - \pi \ln \pi + \pi \ln 4} \)  \( (d) \frac{4 - \pi \ln \pi + \pi \ln 4}{4 + \pi \ln \pi - \pi \ln 4} \)  \( (e) \) none of these.

8. A block of ice, in the shape of a cube, originally having volume 2,000 cm\(^3\), is melting in such a way that the length of each of its edges is decreasing at the rate of 3 cm/hr. Assuming that the block of ice maintains its cubical shape, find the rate of change of the surface area of the cube at the time the volume is 1728 cm\(^3\). In so doing, sketch a picture of the situation, labelling relevant items, and lay out your work very clearly (as is usually requested).

9. A balloon is rising vertically at the rate of 10 ft/s. An observer is standing on the ground 300 ft horizontally from the point where the balloon was released.

(a) Sketch and label a picture of this situation.

(b) At what rate is the distance between the observer and the balloon changing when the balloon is 400 ft high?

10. A balloon is rising vertically at a constant speed of 5 meters per second. A dog is running along a straight line at 15 meters per second, chasing the balloon, and overshoots it. When the dog passes under the balloon, the balloon is 45 meters above the dog. How fast is the distance between the dog and the balloon increasing three seconds after the dog passes under the balloon?

(a) 12.5 m/s  \( (b) 13 \) m/s  \( (c) 13.5 \) m/s  \( (d) \frac{22}{\sqrt{3}} \) m/s  \( (e) \) none of these.

11. If \( f(x) = 3 + x + e^x \) and \( g = f^{-1} \), then \( g'(4) \) is

(a) 0  \( (b) \frac{1}{2} \)  \( (c) 1 \)  \( (d) \) does not exist  \( (e) \) not enough information given.
12. Suppose $f$ is a function with the property that $f'(x) = \cos(x^2)$. Find $g'(x)$, where $g(x) = f(x^2)$.

(a) $g'(x) = 2x\cos(x^4)$  
(b) $g'(x) = \cos(x^4)$  
(c) $g'(x) = \sin(x^4)$  
(d) undefined  
(e) none of these.

13. Suppose $f$ is a function with the property that $f'(x) = \cos(x^2)$. Find $h'(x)$, where $h(x) = f(\frac{1}{x})$.

(a) $h'(x) = -\frac{1}{x^2}$  
(b) $h'(x) = \cos\left(\frac{1}{x^2}\right)$  
(c) $h'(x) = -\frac{1}{x^2}\cos\left(\frac{1}{x^2}\right)$  
(d) undefined  
(e) none of these.

14. Interpret $\lim_{h \to 0} \left(\frac{e^h - 1}{h}\right)$ as a derivative.

(a) $\frac{d}{dx}(e^0)$  
(b) $\frac{d}{dx}(e^x)$  
(c) $\frac{d}{dh}(e^h)$  
(d) $\frac{d}{dx}(e^x)|_1$  
(e) $\frac{d}{dx}(e^x)|_0$.

15. Interpret $\lim_{h \to 0} \left(\frac{\cos h - 1}{h}\right)$ as a derivative.

(a) $\frac{d}{dx}(\cos 0)$  
(b) $\cos'(x)$  
(c) $\cos'(h)$  
(d) $\cos'(0)$  
(e) $\cos'(1)$.

16. If $s(t) = \begin{cases} (t-1)^2 \cos\left(\frac{1}{t-1}\right) & \text{if } t \neq 1 \\ 0 & \text{if } t = 1 \end{cases}$, then $s'(1)$ is

(a) 0  
(b) $\frac{1}{2}$  
(c) 1  
(d) does not exist  
(e) not enough information given.

17. If $g(x) = \begin{cases} (1 - \sqrt{1 - x^2}) \cos\left(\frac{1}{x}\right) & \text{if } 0 < |x| \leq 1 \\ 0 & \text{if } x = 0 \end{cases}$, then $g'(0)$ is

(a) 0  
(b) $\frac{1}{2}$  
(c) 1  
(d) does not exist  
(e) not enough information given.

18. A television camera is positioned 4,000 ft from the base of a rocket-launching pad. A rocket rises vertically and its speed is 600 ft/sec when it has risen 3,000 ft. How fast is the distance from the television camera to the rocket changing at that moment?

(a) 60 ft/sec  
(b) 360 ft/sec  
(c) 720 ft/sec  
(d) 6,000 ft/sec  
(e) not enough information given.

19. If $x\sin y + \cos 2y = \cos y$, then the value of $\frac{dy}{dx}$ at $(1, \frac{\pi}{2})$ is

(a) $-1$  
(b) 0  
(c) 1  
(d) does not exist  
(e) not enough information given.

20. If $f(x) = x^{\sin x}$ for $x > 0$, then $f'\left(\frac{\pi}{2}\right)$ is

(a) 0  
(b) 1  
(c) $\frac{\pi}{2}$  
(d) does not exist  
(e) not enough information given.

21. If $g(t) = e^t$, then $g'(1)$ is

(a) 0  
(b) 1  
(c) $e$  
(d) does not exist  
(e) not enough information given.

22. If $y = x^{f(x)}$, $x > 0$, for some differentiable function $f$, then $\frac{dy}{dx}$ is

(a) $f(x)x^{f(x)-1}$  
(b) $f'(x)(\ln x)x^{f(x)}$  
(c) $f'(x)(\ln x)x^{f(x)} + f(x)x^{f(x)-1}$  
(d) does not exist  
(e) not enough information given.
23. If \( y = f(x)^x \), for some positive-valued differentiable function \( f \), then \( \frac{dy}{dx} \) is

- (a) \( f(x)^x \ln(f(x)) \)
- (b) \( f(x)^x \ln(f(x)) + xf'(x)f(x)^{x-1} \)
- (c) \( xf'(x)f(x)^{x-1} \)
- (d) does not exist
- (e) not enough information given.

24. Let \( x = 1 + \frac{1}{t^2} \) and \( y = 1 - \frac{3}{t} \). Find an equation for the line in the \( xy \)-plane that is tangent to the curve at \( t = 2 \). Also find the value of \( \frac{d^2y}{dx^2} \) at \( t = 2 \).

25. Find \( \frac{d}{dx} \left( \sqrt{e^{\sqrt{x}}} \right) \).

26. Find \( \frac{d}{dx} \left( \sqrt{e^{\sqrt{t}}} \right) \).

27. Find the derivative of \( \ln(x^{20}) \) two different ways.