Study Technique 4  Try this technique to help you prepare for a test. A couple/few days before the test, look over all the homework. The day/night before the test, look over the homework again, and ask yourself what the key concepts and methods are per question (WITHOUT reworking the questions). Jot this down on a sheet of paper and, when you are done, compare with your solutions to see if you were correct. Any question in which you were on the wrong track, look over your solution to it to make sure you see what the main idea is. This work the day/night before will help get your brain to think faster and bring the material to the front of your brain to help you think faster during the actual test.

Part A is due at the start of class on Wed Sept 17, 2008. Part B is not due.

Answer the following questions in groups of two, but turn in one solution sheet per student. Write neatly and orderly as points will be deducted for messy work. No work shown ⇒ partial/full credit not possible, so show as much work as possible.

PART A

1. Evaluate \( \lim_{x \to -\infty} \frac{e^x^5 + 7\pi x^4 - 4x^3}{3x^5 + 7x^3 - 9x} \).

2. Evaluate \( \lim_{x \to \infty} \frac{5x - 7}{\sqrt{3x^2 + 25}} \). (Hint: for large \( x \), what “degree” has the denominator?)

3. Find constants \( a \) and \( b \) that guarantee that the graph of the function defined by \( f(x) = \frac{ax + \sqrt{5}}{7 - bx} \) will have a vertical asymptote at \( x = 5 \) and a horizontal asymptote at \( y = -3 \).

4. Let \( a \) and \( b \) denote constants and let \( f(x) = \begin{cases} 4x + b & \text{if } x < 0 \\ a & \text{if } x = 0 \\ 5 & \text{if } x > 0. \end{cases} \)

Find values for \( a \) and \( b \) so that \( \lim_{x \to 0} f(x) \) exists and is equal to \( f(0) \). Give a coherent mathematical argument to justify your values for \( a \) and \( b \).

PART B (not due)

Below are practice questions for Test 1 on Wednesday. Test 1 will cover college algebra and material in Secs 2.1-2.4 inclusive. The homework is available from my website: www.uta.edu/math/vancliff/T/F08.

1. Find the average rate of change of the function \( f(x) = x^3 \) over the interval \([1, 4]\).

2. Find the exact limit, \( \lim_{x \to 1} f(x) \), if it exists, where \( f \) is defined by \( f(x) = \begin{cases} 2x & \text{if } x \leq 1 \\ 2 & \text{if } x > 1 \end{cases} \); explain.
3. Find the exact limit, \( \lim_{t \to 0} \left( \frac{\sqrt{t+4}-2}{t} \right) \), if it exists; explain.

4. Find the exact limit, \( \lim_{x \to 2} \left( \frac{\sqrt{x+2}-2}{x-2} \right) \), if it exists; explain.

5. Find the exact limit, \( \lim_{t \to 0} \left( \frac{\sin t}{\sqrt{t}} \right) \), if it exists; explain.

6. Find the exact limit, \( \lim_{x \to 2} \left( \frac{\sin(x-2)}{\sqrt{x-2}} \right) \), if it exists; explain.


8. If \( f(x) = \frac{\sqrt{x^2-9}}{2x-6} \), then \( \lim_{x \to 3^+} f(x) \) is (a) \(-\infty\) (b) 0 (c) \(\frac{\sqrt{6}}{2}\) (d) \(\sqrt{6}\) (e) \(\infty\).

9. If \( p(s) = \frac{4-\sqrt{s}}{s-16} \), then \( \lim_{s \to 16} p(s) \) is (a) \(-\infty\) (b) \(-\frac{1}{8}\) (c) 0 (d) \(\frac{1}{8}\) (e) \(\infty\).

10. If \( s(v) = \frac{v^2+2v-8}{v^4-16} \), then \( \lim_{v \to 2} s(v) \) is (a) \(-\infty\) (b) \(\frac{3}{16}\) (c) 1 (d) \(\frac{3}{2}\) (e) \(\infty\).

11. If \( f(x) = \begin{cases} \sqrt{-x} & \text{if } x < 0 \\ 4-x & \text{if } 0 \leq x < 4 \\ (x-4)^2 & \text{if } x > 4 \end{cases} \), then \( \lim_{x \to 0} f(x) \) is (a) 0 (b) \(\frac{1}{2}\) (c) 1 (d) does not exist (e) not enough information given.

12. If \( f(x) = \begin{cases} \sqrt{-x} & \text{if } x < 0 \\ 4-x & \text{if } 0 \leq x < 4 \\ (x+4)^2 & \text{if } x > 4 \end{cases} \), then \( \lim_{x \to 4^-} f(x) \) is (a) 0 (b) 2 (c) 64 (d) does not exist (e) not enough information given.

13. If \( u(t) = \frac{|t-7|}{t-7} \), then \( \lim_{t \to 7^-} u(t) \) is (a) \(-\infty\) (b) \(-1\) (c) 0 (d) 1 (e) \(-7\).

14. Let \( f(x) = 3x + 2 \). Given \( E > 0 \), the largest value of \( d > 0 \) such that \(|x-1| < d \Rightarrow |f(x) - 5| < E\) is (a) \(\frac{E}{3}\) (b) \(\frac{E}{5}\) (c) \(3E\) (d) \(5E\) (e) none of these.

15. Write out a justification of your answer to the previous question.

16. Let \( f(x) = 7x^2 \). Simplify \( \frac{f(x+h) - f(x)}{h} \), eliminating the \( h \) from the denominator.

17. Let \( f(x) = 7x^2 \). Find \( \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \).

18. Find \( \lim_{x \to 0} g(x) \) in order to have \( \lim_{x \to 0} \left( \frac{5-g(x)}{x} \right) = 2 \).

19. Find \( \lim_{x \to 0} g(x) \) in order to have \( \lim_{x \to -4} [x \lim_{x \to 0} g(x)] = 3 \).

20. Go over all assigned homework, worksheet questions and quiz questions.

Test 1 from Fall 2007 is on the next 2 pages. Try working it through in a timed environment (30 minutes) without your book or notes.
You may use a simple calculator (as described on first-day handout). Request paper for rough work.

You have 30 minutes. Keep your eyes on your own work!!

PART A  Show all work. No work \(\Rightarrow\) no partial credit possible.

1. \([Q1, W3:B11]\) [7 points]

Let \(f(x) = 3x^2\) and let \(h\) denote a nonzero number. Simplify \(\frac{f(x + h) - f(x)}{h}\) eliminating \(h\) from the denominator.

2. \([\S2.2: 31]\) [10 points] Find \(\lim_{x \to 1} \frac{x - 1}{\sqrt{x + 3} - 2}\), if it exists.
3. [§2.4:23] [1 point] Find \( \lim_{x \to 0} \frac{\sin 4x}{9x} \), if it exists.

(a) \( \frac{9}{4} \)  (b) 4  (c) 9  (d) \( \frac{4}{9} \)  (e) 1  (f) does not exist.

4. [§2.1,§2.4] [2 points/part] Let \( f(x) = \begin{cases} 
    x^2 + 2x & \text{if } x \leq 1 \\
    2x & \text{if } x > 1 
\end{cases} \) and \( g(x) = \begin{cases} 
    2x^3 & \text{if } x \leq 1 \\
    3 & \text{if } x > 1 
\end{cases} \).

(i) Find \( \lim_{x \to 1^-} f(x) \)  (a) 3  (b) 2  (c) 1  (d) 0  (e) \(-1\)  (f) does not exist.

(ii) Find \( \lim_{x \to 1^+} f(x) \)  (a) 3  (b) 2  (c) 1  (d) 0  (e) \(-1\)  (f) does not exist.

(iii) Find \( \lim_{x \to 1^-} g(x) \)  (a) 3  (b) 2  (c) 1  (d) 0  (e) \(-1\)  (f) does not exist.

(iv) Find \( \lim_{x \to 1^+} g(x) \)  (a) 3  (b) 2  (c) 1  (d) 0  (e) \(-1\)  (f) does not exist.

(v) Find \( \lim_{x \to 1} [f(x) \cdot g(x)] \)  (a) 3  (b) 2  (c) 6  (d) 5  (e) \(-3\)  (f) does not exist.

5. [§2.3] [2 points] Let \( f(x) = 5x + 7 \). Given \( E > 0 \), the largest value of \( d > 0 \) such that

\[ |x - 2| < d \Rightarrow |f(x) - 17| < E \]

is

(a) \( 7E \)  (b) \( 5E \)  (c) \( \frac{E}{7} \)  (d) \( \frac{E}{2} \)  (e) \( \frac{E}{5} \)  (f) \( \frac{E}{3} \).