Due at the start of class on Tues Feb 17, 2004.

Answer the following questions in groups of two or three. Turn in one solution sheet per group. Write the names of your group’s members at the top of the first page of your solution sheet.

1. This question compares the growth of exponential and polynomial functions, and logarithmic and root functions.

   (a) An exponential function such as $f(x) = 2^x$ increases extremely rapidly for large values of $x$, more rapidly than any polynomial function. Using a graphing calculator or similar, show that the graphs of $f(x) = 2^x$ and $g(x) = x^2$ intersect three times. The intersection points divide the $x$-axis into four regions. Describe which function is greater than the other, relative to each region.

   (b) A logarithmic function such as $r(x) = \ln x$ increases extremely slowly for large values of $x$, more slowly than a function like $s(x) = \sqrt[3]{x}$. Using a graphing calculator or similar, sketch graphs of both functions on the same coordinate system for $x > 0$, and determine how many times the two graphs intersect. Describe which function is greater than the other, relative to the regions determined by the intersection points.

2. True or false? For those which are true only if the domain is restricted, give the domain; for those that are false, give the correct equality.

   (a) $\log_b(xy) = (\log_b x)(\log_b y)$
   (b) $\log_b(x + y) = \log_b x + \log_b y$
   (c) $\frac{1}{2} \log_b x = \log_b \sqrt{x}$
   (d) $\log_b \left(\frac{x}{y}\right) = \log_b x + \log_b y$
   (e) $\log_{10} e = (\ln 10)^{-1}$
   (f) $(\log_b x)^r = r \log_b x$
   (g) $y = b^x \iff \log_b y = x$
   (h) $b^{\log_b(Texas)} = Texas$

3. Solve the following for $x$.

   (a) $\log_x(1 - x) = 2$
   (b) $\log_2 x = \log_4 5 + 3\log_2 3$
   (c) $\ln x + \ln(x - 2) = 0$. 
4. Based on the estimate that there are 10 billion acres of arable land on the Earth, and that each acre can produce enough food to feed 4 people, some demographers believe that the Earth can support a population of no more than 40 billion people. The population of the Earth reached approximately 5 billion in 1986 and 6 billion in 1999. If the population of the Earth is growing according to the formula

\[ P(t) = P_0 e^{rt}, \]

where \( t \) is the time after the population is \( P_0 \) and \( r \) is the growth rate, when will the population reach the theoretical limit of 40 billion?

5. If the world population is now 5.8 billion people, and if it continues to grow according to the formula in Question 4 above, with an annual rate of 1.14\%, how long (to the nearest year) will it take before there is only 1 square yard of land per person? (The Earth contains approximately \( 1.68 \times 10^{14} \) square yards of land and 1 billion = \( 10^9 \).)