Due Mon April 14, 2008.
Answer the following questions in groups of two, but turn in one solution sheet per student. Write neatly and orderly as points will be deducted for messy work. No work shown ⇒ partial/full credit not possible, so show as much work as possible.

1. For θ ∈ (−π/2, π/2), consider the graph of \( r = \tan \theta \) (page 714).
   
   (a) Write \( x \) and \( y \) as functions of \( \theta \) only (no \( r \)).
   
   (b) Explain (and so justify) the graph’s behavior as \( \theta \) approaches \( ±\pi/2 \).

2. Let \( f \) denote a continuous function of \( \theta \). Given the two graphs \( r = f(\theta) \) and \( r = 2f(\theta) \), where \( \theta \in [\alpha, \beta] \) for both graphs, are the lengths of the graphs related? Justify.

3. Let \( f \) denote a continuous function of \( \theta \). Consider revolving the two graphs \( r = f(\theta) \) and \( r = 2f(\theta) \), where \( \theta \in [\alpha, \beta] \) for both graphs, around the \( x \)-axis to generate surfaces of revolution. Are the surface areas of these surfaces related? Justify.

4. The area of the region that lies inside the cardioid curve \( r = 1 + \cos \theta \) and outside the circle \( r = \cos \theta \) is not \( \frac{1}{2} \int_{0}^{2\pi} [(1 + \cos \theta)^2 - \cos^2 \theta] \, d\theta \); why not?

5. If \( f \) is a continuous function, the average value of the polar coordinate \( r \) over the curve \( r = f(\theta) \), where \( \theta \in [\alpha, \beta] \), with respect to \( \theta \), is given by
   
   \[ \text{average value of } r = \frac{1}{\beta - \alpha} \int_{\alpha}^{\beta} f(\theta) \, d\theta. \]
   
   Use this formula to find the average value of \( r \), with respect to \( \theta \), over the following graphs:
   
   (a) the cardioid \( r = a(1 - \cos \theta) \), where \( a > 0 \) and constant;
   
   (b) the circle \( r = a \), where \( a > 0 \) and constant.