Check above box for location etc of Final Exam.

Free Math Clinic Review is (check board in math clinic)

The sections covered on the Final Exam are the same as those covered on the homework & worksheets; see my webpage.

Not passing the Final Exam will prevent you from earning a grade of C or higher in the course.

Padmini and I are available in our office hours (see 1st-day handout). I will also be available Friday May 2, 2:30-3:30 pm, in PKH 462, in case you have questions. The Math Clinic will hold a review session (see above).

This worksheet contains practice questions that are intended to help you identify any gaps in your understanding; they do NOT form a model for the Final Exam. Moreover, you should review any homework problems you have not already done & make sure you understand all the homework well, the worksheets, tests & quizzes well at least a few days before the Final Exam. In the last 48 hours before the Final Exam, reread ALL the homework problems including the worksheet questions, skim through the lecture notes, & go over the quizzes & tests and these practice questions again.

Learn some basic trigonometric substitutions, quadratic formula, pythagorean formula & simple formulae from this class. Put on your 3″ × 5″ card the more complicated formulae first, and, if still room, other formulae (or an example of a problem you find difficult to recall).

Bring with you to the Final Exam a working calculator (with working batteries!), satisfying the criteria on the first-day handout and a 3″ × 5″ card with hand-written formulae. Cell phones should be out of sight and switched off (not merely silent).

If you wish to leave the room during the Final Exam, you should ask permission first & turn in your Final Exam to me. Only in exceptional circumstances will I let you continue the Final Exam should you return. (So it is better to be 3 minutes late to the Final Exam, rather than ask to go to the bathroom during the Final Exam.) If you finish early but prefer to stay in the room, then you should NOT get out any work, book nor item, no matter what the subject matter is. Should you wish to leave the Final Exam early, then you may.

It is your responsibility to be on time.

If you wish to know your grade early, turn in to me a STAMPED ADDRESSED postcard (or a postcard inside a stamped addressed envelope) with the following listed on the back (OR send me an e-mail that includes the following):

(a) Honr/Math 2425
(b) Your full name written clearly & MAV ID number
(c) Grade on Final Exam =
(d) Grade in class =
1. Rework the practice questions provided for Tests 1-3 (W3, W7, W11). Go over all worksheet and quiz questions and Tests 1-3.

2. Find \( \int \frac{1}{\sqrt{2t^2}} dt \).

3. Find \( \int x^4 \ln x \, dx \).

4. Find \( \int \frac{1}{x^2 - 3x} \, dx \).

5. Find \( \int \frac{x + 5}{x^2 - 3x} \, dx \).

6. Find \( \int \frac{1}{x(x-3)^2} \, dx \).

7. Determine \( \int_0^3 \frac{1}{(3x - 8)^{4/3}} \, dx \).

8. The correct trigonometric substitution for the integration \( \int \sqrt{9 - x^2} \, dx \) is

   (a) \( x = 9 \sin \theta \)  (b) \( x = 3 \sec \theta \)  (c) \( x = 3 \tan \theta \)  (d) \( x = 3 \sin \theta \)  (e) \( x = 9 \sec \theta \).

9. To compute \( \int \sqrt{4 - t^2} \, dt \) by using integration by substitution, one may use

   (a) \( t = 4 \sin \theta \)  (b) \( t = 2 \sin \theta \)  (c) \( t = \sqrt{2} \sin \theta \)  (d) \( t = \sin 2 \theta \)  (e) \( t = \sin 4 \theta \).

10. Compute \( \int_0^1 xf''(5x) \, dx \) given that \( f(0) = 0 \), \( f'(0) = 3 \), \( f(5) = 10 \), \( f'(5) = 7 \) (where \( f' \) denotes the derivative of \( f \)).

    (a) 1  (b) \( \frac{9}{5} \)  (c) \(-3\)  (d) \(-215\)  (e) cannot be determined from the information given.

11. Let \( F(x) = \int f(x) \, dx \) (i.e., \( F'(x) = f(x) \)). Which of the following are true?

    I. \( \int f(x) \sin x \, dx = -f(x) \cos x + \int f'(x) \cos x \, dx \)

    II. \( \int f(x) \sin x \, dx = F(x) \sin x - \int F(x) \cos x \, dx \).

    (a) I only  (b) II only  (c) both I & II  (d) neither I nor II  (e) the answer cannot be determined from the information given.

12. You are given that \( xF(x) = \int f(x) \, dx \). Find \( \int f(x) \ln x \, dx \).

    (a) \( xF(x) \ln x - f(x) \)

    (b) \( F(x) - \int \ln x \, dx \)  (c) \( F(x) \ln x \)  (d) \( xF(x) \ln x - \int F(x) \, dx \)  (e) \( f(x) + \frac{1}{x} \).

13. Find the limit of the sequence \( a_n = \sqrt{n^2 - 1} - \sqrt{n^2 + n} \). (Hint: multiply by \( \frac{1}{n} \) and use L'Hôpital's Rule).

14. Find the limit of the sequence \( a_n = n - \sqrt{n^2 - n} \). (Hint: use the same method as in the preceding question.)
15. Find the radius of convergence \( R \) for the series \( \sum_{k=0}^{\infty} (2x - 3)^k \).

(a) \( R = 1 \)  
(b) \( R = \frac{1}{2} \)  
(c) \( R = 0 \)  
(d) \( R = \infty \)  
(e) \( R = 2 \).

16. Find the radius of convergence and the interval of convergence of the series \( \sum_{n=0}^{\infty} \frac{(-1)^n(3x - 2)^n}{(n + 2)! 5^n} \).

17. Find the radius of convergence and the interval of convergence of the series \( \sum_{n=0}^{\infty} \frac{(-1)^n(3x - 2)^n}{(n + 2)5^n} \).

18. Which one of the following is the second-order Taylor polynomial for the function \( f(x) = \ln x \) centered at \( x = e \)?

(a) \( (x - 1) - \frac{1}{2}(x - 1)^2 \)  
(b) \( \ln x + \frac{1}{x} (x - e) - \frac{1}{2x^2} (x - e)^2 \)  
(c) \( 1 + \frac{1}{e} (x - e) - \frac{1}{2e^2} (x - e)^2 \)  
(d) \( (x - e) - \frac{1}{2} (x - e)^2 \)  
(e) \( 1 + \frac{1}{e} (x - e) - \frac{1}{e^2} (x - e)^2 \).

19. Use the definition of Maclaurin series to determine the Maclaurin series for \( f(x) = \frac{1}{x+a} \) (where \( a \) is a nonzero constant), and find its interval of convergence.

20. Find the area of the region enclosed by the graph of \( y = x^2 \) and the graph of \( y = 2x - x^2 \).

(a) \( \frac{1}{2} \)  
(b) \( \frac{1}{3} \)  
(c) \( \frac{1}{4} \)  
(d) 1  
(e) \( \frac{2}{3} \).

21. For all \( n > 0 \), find the area between the curves \( y = x^n \) and \( y = x^{n+1} \) on \([0, 1]\).

(a) \( \frac{1}{n+1} - \frac{1}{n+2} \)  
(b) \( \frac{1}{n+2} - \frac{1}{n+1} \)  
(c) \( \frac{1}{n} - \frac{1}{n+1} \)  
(d) \( \frac{1}{6} \)  
(e) \( \frac{1}{12} \).

22. Set up an integral that gives the length of the graph of \( y = x^\frac{3}{2} \) on \([0, 4]\).

23. Find the length of the polar curve \( r = \theta^2 - 1 \) for \( \theta \in [0, 3] \).

24. Set up an integral that gives the area of the surface generated by revolving the curve \( y = 2\sqrt{x} \), for \( x \in [1, 2] \), about the \( x \)-axis.

25. Which one of the following integrals gives the area inside the polar curve given by \( r = 2 + \sin \theta \)?

(a) \( \int_{0}^{2\pi} (2 + \sin \theta) \, d\theta \)  
(b) \( \int_{0}^{\pi} (2 + \sin \theta)^2 \, d\theta \)  
(c) \( \frac{1}{2} \int_{0}^{2\pi} (2 + \sin \theta)^2 \, d\theta \)  
(d) \( \frac{1}{2} \int_{0}^{2\pi} (2 + \sin \theta) \, d\theta \)  
(e) \( \frac{1}{2} \int_{-\pi/2}^{\pi/2} (2 + \sin \theta)^2 \, d\theta \).
26. Let \( B \) denote the region enclosed by the graphs of \( y = x^3 + 3, \ y = 2 \) and \( x = 2 \). Set up an integral that represents the volume of the solid generated when the region \( B \) is revolved about the horizontal line \( y = 1 \).

27. Let \( B \) denote the region enclosed by the graphs of \( y = \sqrt{x + 4}, \ y = 2 \) and \( x = 21 \). Set up an integral that represents the volume of the solid generated when the region \( B \) is revolved about the horizontal line \( y = 1 \).

28. Find the distance between the points \( (1, 4, 8) \) and \( (4, -2, 11) \) in \( \mathbb{R}^3 \).

29. Find the unit vector that points in the direction of the vector \( \mathbf{PQ} \) from \( P(1, 2, -3) \) to \( Q(4, 2, -7) \).
   (a) \( \langle 3, 0, 4 \rangle \) (b) \( \langle -\frac{3}{5}, 0, \frac{4}{5} \rangle \) (c) \( \langle \frac{3}{5}, 0, -\frac{4}{5} \rangle \) (d) \( \langle \frac{3}{5}, 0, \frac{4}{5} \rangle \) (e) \( \langle -\frac{3}{5}, 0, -\frac{4}{5} \rangle \).

30. To which of the following vectors is \( 2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k} \) orthogonal?
   (a) \( \mathbf{j} \) (b) \( 2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k} \) (c) \( \frac{1}{2}\mathbf{i} + \frac{3}{2}\mathbf{j} - \frac{1}{4}\mathbf{k} \) (d) \( \mathbf{i} \) (e) \( 2\mathbf{i} + \mathbf{k} \).

31. Find the value(s) of \( x \) such that the vectors \( \mathbf{u} = \langle x^2, -1, 2 \rangle \) and \( \mathbf{v} = \langle 1, x, -1 \rangle \) are orthogonal.
   (a) \( x = 1, x = 2 \) (b) \( x = -1, x = -2 \) (c) \( x = 1, x = -2 \) (d) \( x = -1, x = 2 \) (e) \( x = -1 \) only.

32. If \( \mathbf{v} \) and \( \mathbf{w} \) are specific nonzero vectors in \( \mathbb{R}^3 \) that satisfy \( \mathbf{v} \cdot \mathbf{w} = |\mathbf{v} \times \mathbf{w}| \), what is the angle between \( \mathbf{v} \) and \( \mathbf{w} \)?
   (a) \( 0 \) (b) \( \frac{\pi}{4} \) (c) \( \frac{\pi}{2} \) (d) \( \frac{3\pi}{4} \) (e) \( \pi \).

33. Find an equation, in standard form, for the sphere that passes through the origin and is centered at the point \( (-1, 3, 2) \).

34. Find the center & radius of the sphere given by the equation \( x^2 + 2x + y^2 + 6y + z^2 + 14z + 59 = 0 \).

35. Find the vector projection of \( \mathbf{u} \) onto \( \mathbf{v} \) where \( \mathbf{u} = \mathbf{i} - 2\mathbf{j} + 3\mathbf{k} \) and \( \mathbf{v} = 4\mathbf{i} + 5\mathbf{j} - \mathbf{k} \).
   (a) \( \mathbf{i} - \mathbf{j} + \mathbf{k} \) (b) \( -\frac{36}{12} \mathbf{i} - \frac{45}{12} \mathbf{j} + \frac{9}{12} \mathbf{k} \) (c) \( -\frac{36}{12} \mathbf{i} - \mathbf{j} + \frac{9}{12} \mathbf{k} \)
   (d) \( \frac{36}{12} \mathbf{i} + \frac{45}{12} \mathbf{j} - \frac{9}{12} \mathbf{k} \) (e) \( -\frac{36}{12} \mathbf{i} - \frac{45}{12} \mathbf{j} + \frac{9}{12} \mathbf{k} \).

36. Let \( \mathbf{v} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k} \) and \( \mathbf{w} = -\mathbf{i} + 4\mathbf{j} + 2\mathbf{k} \).
   (a) Calculate \( \mathbf{v} \times \mathbf{w} \). (b) Show that \( \mathbf{v} \times \mathbf{w} \cdot (r\mathbf{v} + s\mathbf{w}) = 0 \) for all \( r, s \in \mathbb{R} \).

37. Find an equation for the plane that passes through the point \( (1, 1, 1) \) and is parallel to the plane \( 4x + 2y - 7z + 12 = 0 \).
   (a) \( 4x + 2y - 7z + 2 = 0 \) (b) \( 4x - 2y - 7z + 5 = 0 \) (c) \( 4x - 2y - 7z - 9 = 0 \)
   (d) \( 4x - 2y + 7z - 9 = 0 \) (e) \( 4x + 2y - 7z + 1 = 0 \).

38. Find an equation for the plane that passes through the points \( (3,2,1), \ (2,1,-1) \) and \( (-1,3,2) \).

39. Find the points of intersection of the line given by \( x = 6 - 2t, \ y = 1 + t, \ z = 3t \) with each of the coordinate planes.

40. Parametrize the line of intersection of the two planes \( x - y + z = 1 \) and \( x + 2y - 3z = 1 \).