Study Technique 3 After you complete your homework, read the section(s) in the textbook that will be covered in the next lecture. It is likely that you will not understand everything you read; however, reading the sections before lecture will help you understand what is presented in the lecture.

Due at the start of lecture (not lab) on Thursday Feb 12, 2009.

Answer the following questions in groups of two or three; turn in one solution sheet per student. Write neatly and orderly as points will be deducted for messy work. No work shown ⇒ partial/full credit not possible, so show as much work as possible.

PART A Sketch the graph of a function $f$ that satisfies the given conditions.

1. $f$ has domain $(-\infty, \infty)$, $\lim_{x \to 4^-} f(x) = -1$, $\lim_{x \to 4^+} f(x) = 1$ and $f(4) = 0$.

2. $f$ has domain $(-\infty, \infty)$, $\lim_{x \to 4^-} f(x) = -1$, $\lim_{x \to 4^+} f(x) = 1$ and $f(4) = 1$.

3. $f$ has domain $(-\infty, \infty)$, $\lim_{x \to 2^-} f(x) = -\infty$, $\lim_{x \to 2^+} f(x) = \infty$, $\lim_{x \to \infty} f(x) = 1$ and $\lim_{x \to -\infty} f(x) = 0$.

PART B

4. Let $a$ and $b$ denote constants and let $f(x) = \begin{cases} 4x + b & \text{if } x < 0 \\ 5 & \text{if } x = 0 \\ \frac{\sin ax}{x} & \text{if } x > 0. \end{cases}$

Find values for $a$ and $b$ so that $\lim_{x \to 0} f(x)$ exists and is equal to $f(0)$. Give a coherent mathematical argument to justify your values for $a$ and $b$.

5. Find $\lim_{x \to -\infty} \frac{e^{x^5} + 7\pi x^4 - 4x^3}{3x^5 + 7x^3 - 9x}$.

6. Find $\lim_{x \to \infty} \frac{5x - 7}{\sqrt{3x^2 + 25}}$. (Hint: for large $x$, what “degree” has the denominator?)

7. Find $\lim_{x \to 3^+} \frac{\sqrt{x^2 - 9}}{2x - 6}$.

8. Let $f(x) = x + 2 + \frac{1}{2x - 6}$. Find equations for the asymptotes of $f$.

9. Find constants $a$ and $b$ that guarantee that the graph of the function $f(x) = \frac{ax + \sqrt{5}}{7 - bx}$ will have a vertical asymptote at $x = 5$ and a horizontal asymptote at $y = -3$. 

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PART C

10. In each case, sketch the graph of a function $f$ that satisfies the given conditions.

(a) $f$ is continuous everywhere except at $x = 4$, at which point $f$ is continuous from the right.

(b) $f$ has domain $(-\infty, \infty)$, $f$ is discontinuous at $x = 4$ and $f(4) = 1$; however, $f$ also has the property that if the value of $f(4)$ were changed from 1 to 0, then $f$ would become continuous at $x = 4$.

11. Regarding the function $f$ in question 4, find all values of $a$ and $b$ so that $f$ will be continuous at $x = 0$. 

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