Due at the start of lecture (not lab) on Thurs Mar 12, 2009.

Answer the following questions in groups of two, but turn in one solution sheet per student. Write neatly and orderly as points will be deducted for messy work. No work shown ⇒ partial/full credit not possible, so show as much work as possible.

PART A: Parametrizations

Example (Page 195 in textbook)

The position \((x, y)\) of a particle moving in the \(xy\)-plane is given by the parametrization
\[
x = \sqrt{t}, \quad y = t, \quad t \geq 0.
\]

Identify the path traced by the particle and describe the motion of the particle.

Solution

We try to identify the path by eliminating \(t\). With luck, this will produce an algebraic relation between \(x\) and \(y\) that we recognize. In this example, we obtain
\[
y = t = (\sqrt{t})^2 = x^2,
\]
which we recognize as a parabola through the origin.

It would be a mistake to conclude that the particle’s path is the entire parabola; in fact, it is only half. We see this by plotting the (initial) point \((x(0), y(0)) = (0, 0)\) and recognizing that \(\sqrt{t}\) is positive for all \(t \geq 0\). We can also see this by plotting a few points: if \(t = 1\), then \((x(1), y(1)) = (1, 1)\); if \(t = 4\), then \((x(4), y(4)) = (2, 4)\); if \(t = 9\), then \((x(9), y(9)) = (3, 9)\). Typically, when we plot points, we label them with the \(t\)-value, as indicated below. Note that a combination of the domain of \(t\) and the form of \(x(t)\) prohibit any points being on the left-hand side of the parabola!

Hence, the path traced out is that of the parabola \(y = x^2\). The motion of the particle starts at the origin \((0, 0)\) at \(t = 0\) and, as \(t\) increases, traces out the right-hand side of the parabola. Arrows on the graph indicate the direction of increase of \(t\).

In the preceding example, \(t\) was the parameter for the curve, and the domain of \(t\) was the parameter interval. If the parameter interval is \([a, b]\), then \((x(a), y(a))\) is the initial point and \((x(b), y(b))\) is the terminal point.

1. Let \(x = t^2 - 2t, \ y = t + 1, \ \text{where} \ t \in \mathbb{R}\).

   (a) Sketch the curve defined by these parametric equations by completing the table below and plotting the points \((x, y)\) determined by the values in the table. Plot the points in the order listed in the table and connect them, drawing arrows to indicate the direction of your curve.

   \[
   \begin{array}{c|cccccc}
   t & -2 & -1 & 0 & 1 & 2 & 3 & 4 \\
   \hline
   x &  \quad &  \quad &  \quad &  \quad &  \quad &  \quad \\
   y & \quad &  \quad &  \quad &  \quad &  \quad &  \quad &  \\
   \end{array}
   \]

   A particle whose position is given by these parametric equations will travel along the curve you just drew in the direction of your arrows. Note that the points are not equally spaced on the curve; it appears that the particle slows down and then speeds up as \(t\) increases.
(b) Solve the \( y \)-formula for \( t \) and substitute for \( t \) in the \( x \)-formula to obtain an equation involving only \( x \) and \( y \) (not \( t \)). Use your equation to identify the curve you sketched in (a).

2. Five parametric curves are given below; sketch each one. (Note: if you eliminate the parameter \( t \), you will find that each has the same equation in \( x \) and \( y \). However, the parametric curves might be different.)

(a) \( x = t, \ y = t^2, \ t \in \mathbb{R} \). (b) \( x = t^2, \ y = t^4, \ t \in \mathbb{R} \). (c) \( x = \sin t, \ y = \sin^2 t, \ t \in \mathbb{R} \).
(d) \( x = \cos t, \ y = \cos^2 t, \ t \in \mathbb{R} \). (e) \( x = e^t, \ y = e^{2t}, \ t \in \mathbb{R} \).

3. Suppose that a coyote fires a missile at a roadrunner from 500 miles away and that the missile follows a flight path given by the parametric equations
\[
\begin{align*}
    x &= 100t, \\
    y &= 16t(5-t), \quad \text{where} \quad t \in [0, 5] \text{ in minutes.}
\end{align*}
\]
Two minutes later, the roadrunner (still in the same location) fires an interceptor missile that follows the flight path
\[
\begin{align*}
    x &= 100(9-2t), \\
    y &= 24(t-2)(7-t), \quad \text{where} \quad t \in [2, 7] \text{ in minutes.}
\end{align*}
\]
Determine whether or not the interceptor missile hits its target.

**PART B:** Chain Rule

4. Functions \( f, g, \) and \( h \) are continuous and differentiable for all real numbers, and some of their values, and values of their derivatives, are given by the table below.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
<th>( g(x) )</th>
<th>( h(x) )</th>
<th>( f'(x) )</th>
<th>( g'(x) )</th>
<th>( h'(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>-1</td>
<td>-1</td>
<td>4</td>
<td>1</td>
<td>-3</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>3</td>
<td>0</td>
<td>2</td>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>-2</td>
<td>1</td>
<td>3</td>
</tr>
</tbody>
</table>

(a) If \( F(x) = g(x)e^{2f(x)} \), find \( F'(0) \).
(b) If \( G(x) = g(h(x)) \), find \( G'(2) \).
(c) If \( H(x) = e^{h(x)+g(x)^2} \), find \( H'(1) \).