POWER INDEX RANKINGS FOR COOPERATIVE GAMES

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For a given cooperative game in TU form, $v : 2^N \to \mathbb{R}$, every solution $\psi : G \to \mathbb{R}^N$ provides a way to rank the players, namely, via the values of $\psi_j(v)$.

We will recall a useful decomposition of the space of games, and with it we will prove the following:

Theorem 1. *Let $\psi : G \to \mathbb{R}^n$ be any linear symmetric solution. Then there exists a unique solution $\phi : G \to \mathbb{R}^n$ which is linear, symmetric and efficient, giving the same power index ranking as ψ. Moreover, if ψ is self-dual then ϕ will also be selfdual.*

Next, we will introduce the concept of a team game. This is a cooperative game which vanishes on every coalition except, perhaps, on those coalitions whose cardinality is some fixed number s.

We will show the following:

Theorem 2. *On team games of fixed cardinality s, every solution gives, essentially, the same power index ranking of the players. Thus, every team game has intrinsically associated a ranking of its players.*

We will give an easy formula to compute this index.