

ALGEBRA SEMINAR

Department of Mathematics
University of Texas at Arlington
presents

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"Torsion in tensor products of modules"

Monday, October 12, 2009
4:00 p.m., 304 Pickard Hall
Refreshments served at 3:45pm

Abstract:

Let $(R; \mathfrak{m}; k)$ be a local integral domain of dimension one, and let M and N be finitely generated R -modules. The tensor product $M \otimes_R N$ usually has torsion, even if both M and N are torsion-free. Of course one can make the tensor product torsion-free by cheating: Take one of the modules to be torsion-free and the other one to be free! An honest example occurs over the ring $C[[T_4; T_5; T_6]]$: Take $M = (T_4; T_5)$ and $N = (T_4; T_6)$, and check that $M \otimes_R N$ is isomorphic to the maximal ideal $(T_4; T_5; T_6)$. Over the ring $C[[T_4; T_5]]$, however, the tensor product of two non-free modules always has torsion. In fact, a 1994 theorem, due to Huneke and me, says that over a hypersurface domain of any dimension, if $M \otimes_R N$ is maximal Cohen-Macaulay, then either M or N is free. (For one-dimensional domains, "maximal Cohen-Macaulay" is the same as "torsion-free".) A related question is the following: If $M \otimes_R M^\alpha$ is maximal Cohen-Macaulay, must M be free? (Here $M^\alpha = \text{Hom}_R(M; R)$.) There are no known counterexamples, and the answer is unknown even for one-dimensional complete intersections. I will discuss these problems, their relation to a celebrated conjecture due to Auslander and Reiten, and the fact that in some sense the pivotal case occurs in dimension one.

Web-Page for Seminars (Department of Mathematics, UTA)
<http://www.uta.edu/math/pages/main/seminar.htm#algebra>

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