The core of ideals

Abstract

Let R be a Noetherian local ring with infinite residue field k and I an R-ideal. The ideal J is a *reduction* of I if $J \subset I$ and $I^{r+1} = JI^r$ for some positive integer r. A reduction can be thought of as a simplification of the ideal I. The notion of a reduction for an ideal was introduced by D. Northcott and D. Rees in order to study multiplicities. Reductions are connected to the study of blowup algebras such as the Rees ring $\mathcal{R}(I) = R[It]$ of I, and the associated graded ring $\text{gr}_I(R) = R[It]/IR[It]$ of I.

In general minimal reductions are not unique. To remedy this lack of uniqueness, one considers the intersection of all reductions, namely the *core* of the ideal, $\text{core}(I)$. This object, that appears naturally in the context of the Briançon-Skoda theorem, encodes information about all possible reductions. We present some recent work on the shape of the core of ideals.