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Title: So you think you understand fractions? Quick, does 23/67 equal 33/97? A mathematician's secret from Euclid to today

Abstract:

What method will someone choose for determining whether or not two fractions are equal?

Expanding to a common denominator requires calculating products, which some people will do, although they will often balk at so much work. Will they more likely grab a calculator and calculate decimal approximations, which may not answer the question. Will some people try reducing the fractions, and if so, what can they conclude with certainty, and on what basis?

I maintain there is a very delicate mathematical issue here, something very powerful but sufficiently subtle that it tends early to become invisible to the mathematician by assuming intuitive second nature, while for others it remains entirely outside conscious awareness, despite perhaps being used occasionally in practice without question.

The modern mathematician will ultimately resolve the issue by appealing to uniqueness of prime factorization, first clearly stated and proven only by Gauss. What can we find in earlier mathematics that sheds light on this question, say in Euclid, or Euler? And what light does it shed on teaching about fractions? Is the biggest secret of fractions too complicated for school or street? Quick, does 23/67 equal 33/97, and why?