

The Forced Non-Linear Schrödinger Equation

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Abstract

We will discuss the forced non-linear Schrödinger equation with a potential, on the half-line,

$$i\frac{\partial}{\partial t}u = -\frac{\partial^2}{\partial x^2}u + Vu + F(x, t, u), \quad x \in (0, \infty), \quad t > 0,$$
$$u(x, 0) = \varphi(x), \quad u(0, t) = f(t), \quad \phi(0) = f(0),$$

where F is the external force (the wavemaker). This equation has many important applications in applied physics; for example, in non-linear optics and in wave propagation in the atmosphere.

We will discuss general properties of the solutions. In particular, the initial-boundary value problem; existence, uniqueness and regularity of local and global solutions; continuity on the external force and the initial data.

Furthermore, we will consider the large-time behavior of the solutions, scattering and inverse scattering, unique reconstruction of the potential and the non-linearity.

The colloquium will be of an introductory nature, and the basic concepts and techniques will be explained in a detailed way.